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NASA TECHNICAL TRANSLATION NASA TT F-14,056

EFFECT OF SONIC BOOM ON BUILDINGS (SECOND REPORT: ELABORATION OF A METHOD FOR CALCULATING THE DEFORMATION OF CONSTRUCTIONS)

Translation of: "Effet du Bang sur les Batiments (2è Rapport: Elaboration d'une méthode de calcul de la déformation des ouvrages)", Centre Scientifique et Technique du Batiment, Paris, December 1970, 63 pages.

(THRU)
N72-11955 (NASA-TT-F-14056) EFFECT OF SONIC BOOM ON BUILDINGS (SECOND REPORT: ELABORATION OF A METHOD FOR CALCULATING THE DEFORMATION OF CONSTRUCTIONS) (Scientific Translation Service) Dec. 1971 69 p CSCL 20A G3/02
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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D. C. 20546 DECEMBER 1971

EFFECT OF SONIC BOOM ON BUILDINGS

(SECOND REPORT: ELABORATION OF A METHOD FOR CALCULATING THE DEFORMATION OF CONSTRUCTIONS) (1)

ABSTRACT. The acoustic response of various room configurations in buildings to sonic booms is calculated. Configurations studied include: single rooms having openings in walls, penetration of booms through flexible walls, two rooms coupled acoustically by openings, rooms with window panes.

The purpose of the present study is to evaluate the deformation of constructions which are sensitive to sonic boom. The deformation is calculated for a ballistic detonation, and the characteristics of the various facades of a building are assumed known. /1*

This study essentially is concerned with vibrations which are produced in interior partitions, ceilings, window panes, which are the most fragile elements in new structures. The determination of these vibrations was done using the classical theory of dynamic deformation of a plate. This theory is applicable in the case of homogeneous partitions and ceilings if there are no substantial internal prestresses. Also, it represents a good approximation for the study of window panes, if these do not have dimensions which are too large.

In order to evaluate the motions of all these elements, it is first necessary to know the propagation of a sonic boom across a building. The boom can propagate through the air. It then penetrates through one or

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*Numbers in the margin indicate pagination in the original foreign text.

(1) Convention No. 69-34-412-00-480-75-01, December 1, 1970

several openings and propagates into the interior. The case of a room having a rectangular opening is studied in the first part.

The sonic boom can also be transmitted through light structural elements (window panes, light partitions). In practice, the overall structure does not have an effect. The transmission of these elements depends on their motions and these two latter phenomena have been studied together in a second part. This takes into account the possibilities of coupling between the panel vibrations and the pressure which they radiate, as in the case of a window with a cavity behind it, for which numerous studies have been made.

1.1 EQUATION OF DYNAMIC EQUILIBRIUM /3

1.1.1 Hypotheses

For an incident standing sinusoidal wave, the wavelength of which is large with respect to the dimensions of the opening, the velocity field in the plane of this opening can be studied in a simple way. In particular, the approximation which consists of assuming that this velocity is uniform can be made. In the same way, if this wavelength is large with respect to the dimensions of the room, the overpressure in it can be considered as uniform, if there are no parasitic sources in the room or at its front, such as walls carrying out vibrations at high frequencies. These hypotheses lead to the classical theory of Helmholtz resonators.

As far as ballistic detonation is concerned, the energy is concentrated primarily at frequencies whose associated wavelengths satisfy the preceding hypotheses, for conventional room dimensions. The velocity field in the opening can therefore be studied in the way described above, to a good degree of approximation. When the sonic boom passes through this opening, it begins to undergo diffraction in the room. After an intermediate time interval, which is short with respect to the fundamental resonance period (calculated with the Helmholtz resonator), during which the overpressure field is not uniform and where the first reflections on the walls are produced, the various incident and reflected waves, which have different but small phase differences, contribute to the establishment of a uniform overpressure in the room. This overpressure is uniform except for the immediate vicinity of the opening where the velocity increase produces a reduction in the vibration amplitude. The variation of the overpressure in the room will be obtained by considering this overpressure to be uniform, except for the beginning of the process.

The difference between the forces applied to the air contained in the opening by the external pressure p_1 and the internal pressure p_2 is equal to the sum of the momentum derivative of the air contained in the throat and the viscosity forces (see Figure 1.1). In general, we can assume that the opening is not too small compared with the wavelengths considered, so that these latter forces can be neglected (see [3]).

1.1.2 Pressure p_i in the room: (rigid walls and non-absorbing walls)

It is assumed that the overpressure is uniform and the room acts like an air volume which satisfies the usual gas laws. If the volume V changes adiabatically, we have

$$\frac{dp_i}{P_0} = -\gamma \frac{dV}{V}$$

, P_0 is the ambient pressure

where:

$$dp_i = -\frac{\rho c^2}{V}\, dV$$

c is the velocity of sound,

ρ the specific mass of the air.

The derivative of the volume with respect to time dV/dt is given by the velocity flux over the periphery of the room. For practically rigid

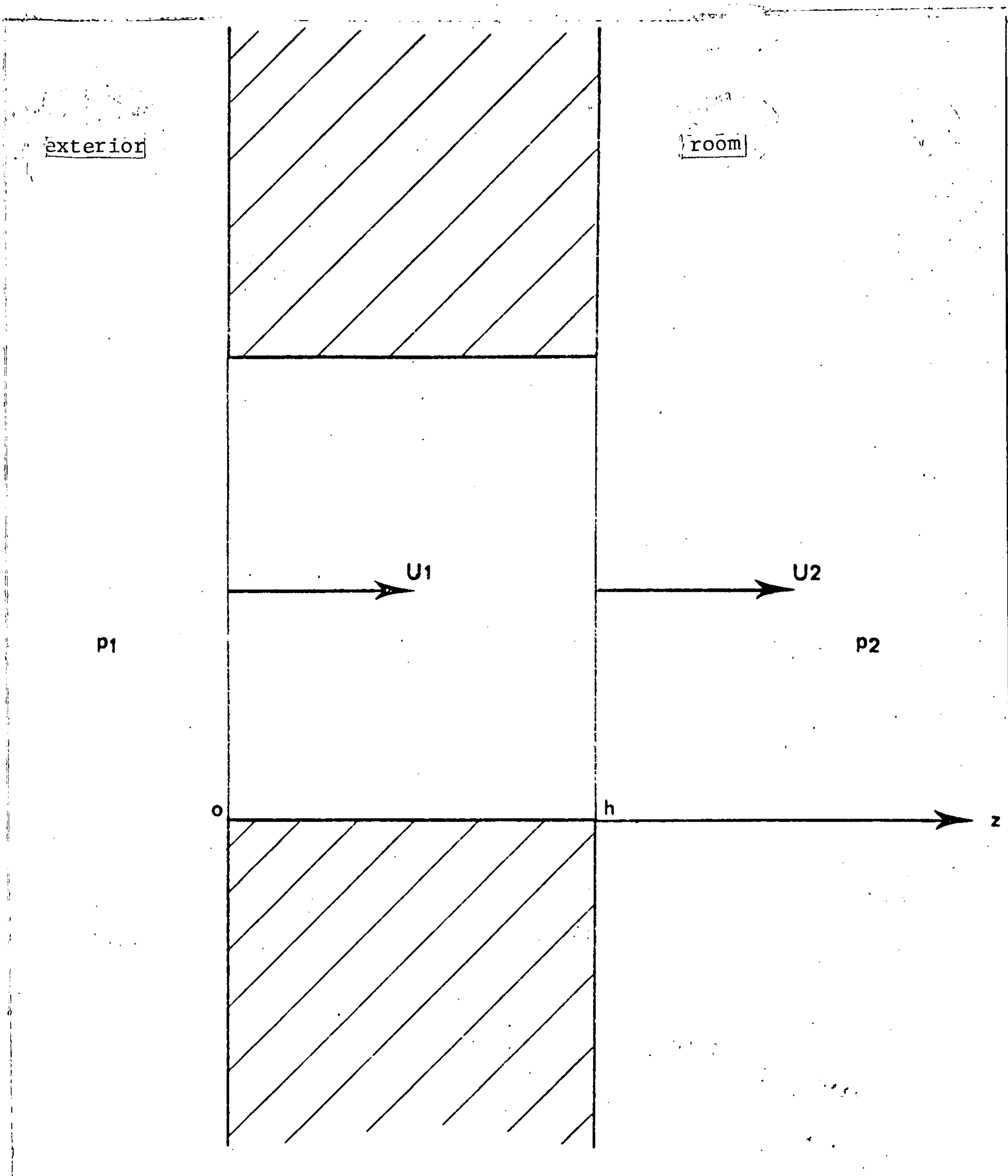


Figure 1.1. Penetration of the sonic boom through an opening.

walls, this flux is reduced to the velocity flux at the opening, from which:

$$\frac{dp_i}{dt} = \frac{\rho c^2}{V} \int_S u_2 \, dS \qquad (1)$$

with S - surface of the opening

u_2 - velocity of a point on the internal side of the opening, oriented from the exterior towards the interior.

1.1.3 Force Applied to the Internal Surface of the Opening /5

Because the acoustic equations are linear, this force will be equal to the sum:

1°) of the pressure force p_i

2°) and the force due to radiation of the air mass which is in motion in the opening. This radiated pressure decreases to zero rapidly when one moves away from the opening. This radiation must result in a zero normal velocity along the walls.

If we know the expression for the radiation (see below), we obtain the following form for this force (oriented positively from the interior towards the exterior of the room):

$$\int_S p_2 \, dS = p_i S + \alpha_2 \int_S \frac{du_2}{dt} dS - \beta_2 \int_S \frac{d^2 u_2}{dt^2} dS$$

1.1.4 Momentum Derivative in the Throat (thickness of the opening)

In order to evaluate this quantity, we can hypothesize that the acoustic perturbation will be propagated in the form of plane waves over the same streamline. In this case (see Figure 1.1)

$$u_2(t) = F(t) - G(t)$$

$$u(t) = F\left(t - \frac{z-h}{c}\right) - G\left(t + \frac{z-h}{c}\right)$$

where:

$$u(t) \# u_2(t) - \frac{z-h}{c}\left[F'(t) + G'(t)\right]$$

The momentum derivative can be approximated by

$$\rho \int_S \left[\int_0^h \frac{du}{dt} dz\right] dS = \rho h \int_S \frac{du_2}{dt} dz + \frac{\rho h^2}{2c} \int_S \left[F''(t) + G''(t)\right] dS$$

$$\rho \int_S \left[\int_0^h \frac{du}{dt} dz\right] dS = \rho h \int_S \frac{du_2}{dt} dS + \frac{h^2}{2c^2} \int_S \frac{d^2 p_2}{dt^2} dS$$ /6

$$\rho \int_S \left[\int_0^h \frac{du}{dt} dz\right] dS \# \rho h \int_S \frac{du_2}{dt} dS + \frac{h^2 S}{2c^2} \frac{d^2 p_i}{dt^2}$$

by eliminating the derivatives of u_2 of the second order.

1.1.5 Forces Exerted on the External Surface of the Opening

If the velocity radiation force at the input of the throat is of the form:

$$-\alpha_1 \int_S \frac{du_1}{dt} dS + \beta_1 \int_S \frac{d^2 u_1}{dt^2} dS$$

we obtain, by superposition as before, a force equal to:

$$\int_S p_1 dS = \int_S p_e dS - \alpha_1 \int_S \frac{du_1}{dt} dS + \beta_1 \int_S \frac{d^2u_1}{dt^2} dS$$

p_e is the pressure (directed from the exterior to the interior of the room) which would prevail on the facade assuming a closed opening (the velocity in this case is zero at the opening level).

By replacing u_1 (t) by:

$$u_1(t) = u_2(t) + \frac{h}{\rho c^2} \frac{dp_2}{dt}$$

we obtain the following expression for the desired force:

$$\int_S p_1 dS \# \int_S p_e dS - \alpha_1 \int_S \frac{du_2}{dt} dS + \beta_1 \int_S \frac{d^2u_2}{dt^2} dS$$
$$- \alpha_1 \frac{hS}{\rho c^2} \frac{d^2p_i}{dt^2} + \beta_1 \frac{hS}{\rho c^2} \frac{d^3p_i}{dt^3}$$

1.1.6 <u>Equation of Equilibrium</u> /7

This equation is written as:

$$\int_S p_e dS = (\alpha_1 + \alpha_2 + \rho h) \int_S \frac{du_2}{dt} dS - (\beta_1 + \beta_2) \int_S \frac{d^2u_2}{dt^2} dS$$
$$+ \left(\alpha_1 \frac{Sh}{\rho c^2} + \frac{h^2 S}{2c^2}\right) \frac{d^2p_i}{dt^2} - \beta_1 \frac{hS}{\rho c^2} \frac{d^3p_i}{dt^3}$$

where, taking (1) into account:

$$p_i + \frac{V}{\rho c^2 S}\left[\alpha_2 + (\alpha_1 + \rho h)\left(1+\frac{Sh}{2V}\right)\right]\frac{d^2 p_i}{dt^2} - \frac{V}{\rho c^2 S}\left[\beta_2 + \beta_1\left(1+\frac{Sh}{2V}\right)\right]\frac{d^3 p_i}{dt^3} = \frac{1}{S}\int_S p_e\,dS$$

1.1.7 Simplification of the Equilibrium Equation

a) First of all, in general we have $\frac{Sh}{2V} \ll 1$

b) The equation obtained is a differential equation of the third order in p_i. The associated characteristic equation in terms of r is of the form:

$$-2KCr^3 + 2LCr^2 + 1 = 0$$

where K, C and L are positive.

The roots are of the form:

$$\begin{cases} r_1 \\ r_2 = r_o + J\Omega \\ r_3 = r_o - J\Omega \end{cases}$$

with

$$\Omega \# \frac{1}{\sqrt{2LC}}$$

The relationships among the roots are written as: /8

$$\begin{cases} r_1(r_o^2 + \Omega_o^2) = \frac{1}{2KC} \\ r_o^2 + \Omega^2 + 2r_o r_1 = 0 \\ 2r_o + r_1 = \frac{L}{K} \end{cases}$$

The first relationship shows that r_1 is positive; the second shows that r_0 is negative. The elimination of r_1 among these two latter relationships results in:

$$\boxed{3r_0^2 - 2r_0\frac{L}{K} - \Omega^2 = 0}$$

and the negative root is:

$$r_0 = \frac{1}{3}\left[\frac{L}{K} - \sqrt{\frac{L^2}{K^2} + 3\Omega^2}\right]$$

In general we have $\frac{L^2}{3K^2\Omega^2} \gg$ (for example 10^3) from which:

$$r_0 \# \frac{1}{3}\frac{L}{K}\left[1 - \left(1 + \frac{3\Omega^2 K^2}{2L^2}\right)\right]$$
$$r_0 \# -\frac{K\Omega^2}{2L}$$

This is as though the roots r_2 and r_3 were solutions of the equation:

$$2LCr^2 + 2KC\Omega^2 r + 1 = 0,$$

which corresponds to the first term of the differential equation

$$\boxed{2LC\frac{d^2p_i}{dt^2} + 2KC\Omega^2\frac{dp_i}{dt} + p_i}$$

c) The solution of (2) is of the form:

$$\boxed{p_{io}(t) + Ae^{r_1 t} + Be^{r_0 t}\sin\Omega t + Ce^{r_0 t}\cos\Omega t}$$

$p_{io}(t)$ is a particular solution,

A, B, C are three constants determined by three conditions.

The solution r_1 is not a physical one and corresponds to the way the coefficients β_1, β_2 are determined (see below) by limited development. In fact, a differential equation having a higher order can be obtained by carrying out the development to a higher order. /9

It can be acknowledged that a particular solution of the differential equation obtained in b) must also satisfy Equation (2).

Assuming this particular solution as the solution $p_{io}(t)$, we therefore obtain of necessity:

A = 0, the overpressure must be zero after an infinite time.

In conclusion, we can substitute a second order differential Equation (3) for the differential Equation (2), which is more physical and must be quite close to reality:

$$\boxed{\frac{V}{\rho c^2 S}\left[\alpha_1+\alpha_2+\rho h\right]\frac{d^2 p_i}{dt^2}+\frac{\beta_2+\beta_1}{\alpha_2+\alpha_1+\rho h}\frac{dp_i}{dt}+p_i=\frac{1}{S}\int_S p_e\,dS} \qquad (3)$$

This differential equation is identical to the one for a simple resonator made up of a mass and a spring.

The system consisting of the room and the opening therefore behaves like a resonator, with the following eigenfrequency:

$$\omega_0 \# c\sqrt{\frac{\rho S}{V(\alpha_1+\alpha_2+\rho h)}}$$

and restoring factor $Q = \pi/S_0$; S_0 is the logarithmic decrement:

$$S_0 = \frac{(\beta_1 + \beta_2)\,\omega_0}{4\pi(\alpha_1 + \alpha_2 + \rho h)}$$

1.2 DETERMINATION OF THE COEFFICIENTS INVOLVED IN THE RADIATION FROM THE OPENING /10

1.2.1 The radiation is determined by the velocity distribution at the level of the opening. This distribution depends not only on the form and dmensions of the opening, but also on the room and even the building. The height of the exterior façade is usually much smaller than the wavelength under consideration, and the velocity distribution is different from the one obtained for an opening in an infinite baffle. In the same way, the influence of the room walls due to a reflected wave which comes toward the opening with velocity vectors which are different from those of the incident wave will modify the velocity distribution. Since there is no way to determine these velocities in practice, known expressions can be used which have been established for the case of openings in an infinite baffle. They will be more applicable, the farther the extremities of the facade or the walls are away from the edges of the opening.

These expressions are calculated for the two following cases: uniform velocity in the opening, and uniform radiation pressure over this opening. The real case will be somewhere in between the two hypothetical cases.

1.2.2 Hypothesis of Uniform Velocity

For a rectangular piston in an infinite baffle, usually the following expression for the radiation is assumed:

$$p_{rad} = \frac{\rho}{2\pi} \int_S \frac{du}{dt}\left(t - \frac{r}{c}\right) \frac{dS}{r}$$

r is the distance between a moving point of the surface S and the point where the pressure is being calculated. u is the velocity at a moving point of the surface.

For a sinusoidal vibration we have:

$$p_{rad} = \frac{j\omega\rho}{2\pi} U \int_S e^{-jkr} \frac{dS}{r}$$

The applied force is therefore: /11

$$F_{rad} = \int_S p_{rad} S = \frac{j\omega\rho}{2\pi} \int_S \left[U \int_S \right.$$

It is given in [1]:

$$F_{rad} = \frac{j\omega\rho U}{2\pi} \left\{ 4 \sum_{n=0}^{\infty} \left[A_{2n} k^{2n} / (2n+2)! \right] - 4jk \sum_{n=0}^{\infty} \left[B_{2n} k^{2n} / (2n+3)! \right] \right\}$$

with:

$$\left\{ \begin{aligned} A_{2n} &= (-1)^n \left\{ L\ell^{2n+2} \int_0^{\mathrm{Arctg}\,\varphi} \frac{d\theta}{\cos^{2n+1}\theta} + \ell L^{2n+2} \int_0^{\mathrm{Arctg}\,\varphi} \frac{d\theta}{\cos^{2n+1}\theta} \right. \\ &\qquad \left. - \left[(L^2+\ell^2)^{n+\frac{3}{2}} - L^{2n+3} - \ell^{2n+3} \right] / (2n+3) \right\} \\ B_{2n} &= (-1)^n \left\{ L\ell^{2n+3} \int_0^{\mathrm{Arctg}\,\varphi} \frac{d\theta}{\cos^{2n+2}\theta} + \ell L^{2n+3} \int_0^{\mathrm{Arctg}\,\varphi} \frac{d\theta}{\cos^{2n+2}\theta} \right. \\ &\qquad \left. - \left[(L^2+\ell^2)^{n+2} - L^{2n+4} - \ell^{2n+4} \right] / (2n+4) \right. \end{aligned} \right.$$

$k = \omega/c$; L = length of the opening; l = width of the opening; $\phi = L/1$.

For usual rooms, we can restrict ourselves to A_{20} and B_{20} in general. Therefore we obtain the following force due to radiation:

$$\boxed{F_{rad} = \frac{\rho S^2}{2(L+\ell)} H \frac{du}{dt} - \frac{\rho S^2}{2\pi c} \frac{d^2 u}{dt^2}}$$

with

$$H = \frac{2}{\pi}\left\{\left(1+\frac{1}{\varphi}\right)\mathrm{Log}\left[(1+\varphi^2)^{\frac{1}{2}}+\varphi\right] + (1+\varphi)\,\mathrm{Log}\left[\left(1+\frac{1}{\varphi^2}\right)^{\frac{1}{2}}+\frac{1}{\varphi}\right]\right.$$
$$\left. + \frac{1}{3}\varphi(1+\varphi)\left[1+\frac{1}{\varphi^3}-\left(1+\frac{1}{\varphi^2}\right)^{3/2}\right]\right\}$$

From the coefficients given in § 1.1 we find:

$$\boxed{\begin{aligned} \alpha &= \frac{\rho S}{2(L+\ell)} H \\ \beta &= \frac{\rho S}{2\pi c} \end{aligned}} \qquad (4)$$

These coefficients result in an eigenfrequency of 8.9 Hz and a damping of 3% for a room having the dimensions 4 x 4.15 x 2.5 and an opening having the thickness 0.30 and the dimensions 1.30 x 1.30.

Conventional rooms and openings must have eigenfrequencies between 5 and 15 Hz.

1.2.3 Uniform Pressure Hypothesis

We can write:

$$\boxed{\int_s u\,dS = A\,p_{rad}}$$

A is the admittance of the system. For wavelengths which are large with respect to the dimensions of the opening, it is equal to the following for the case of the sinusoidal wave [2]:

$$A = \frac{K^2}{2\pi\rho c} - j\frac{K}{\rho\omega}$$

ω is the oscillation under consideration, and K is the conductivity. An expression for it is given in [2]:

$$K = \frac{\pi}{2}\,\frac{L+\ell}{\mathrm{Log}\left[2\left(\varphi^{\frac{1}{2}}+\varphi^{-\frac{1}{2}}\right)\right]}$$

This expression is closer to reality, the farther away the opening is (ϕ_{large}). Therefore it will certainly be valid here and in any case where the velocity field is planer almost everywhere. In particular, we can assume that it is a good approximation in the case where the height of the opening is equal to the height of the room. In practice we can write: /13

$$\frac{P_{rad}\,S}{\int_S u\,dS} \# \frac{\rho\omega^2 S}{2\pi c} + j\omega\rho\frac{S}{K}$$

where, if the wave is not sinusoidal:

$$P_{rad}\,S = \rho\frac{S}{K}\int_S \frac{du}{dt}\,dS - \frac{\rho S}{2\pi c}\int_S \frac{d^2u}{dt^2}\,dS$$

We obtain the coefficients from Equation (3):

$$\alpha = \rho\frac{S}{K}$$

$$\beta = \frac{\rho S}{2\pi c} \qquad (4')$$

These two expressions differ slightly from the expressions (4). They result in an eigenfrequency of 9.1 Hz and a damping of 3.2% for the same room and the same opening considered before.

1.3 RESPONSE TO A N WAVE

The N wave will be assumed as indicated in Figure 1.2.

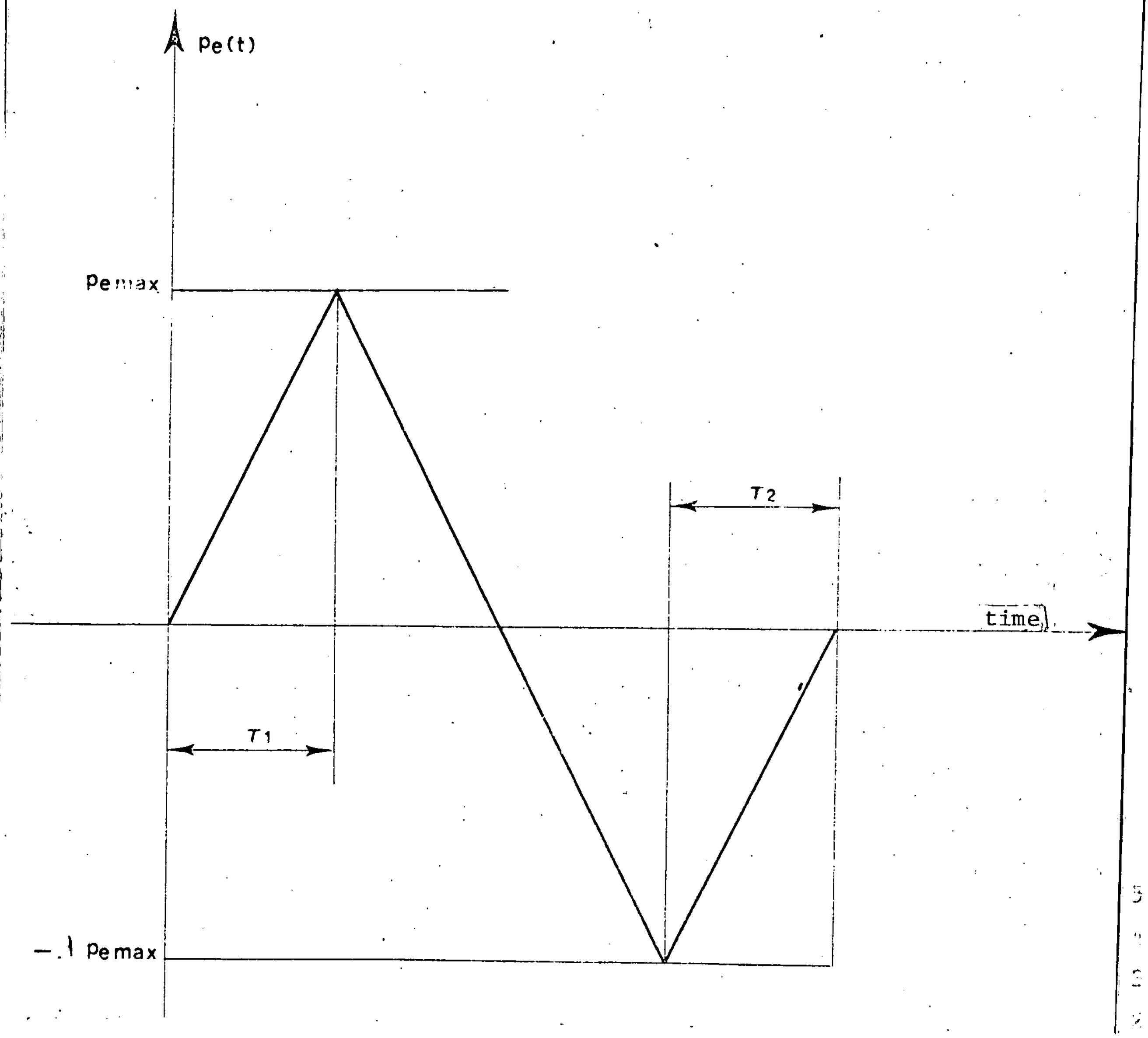


Figure 1.2. Diagram of a "N" wave.

1.3.1 Laplace Transform of the N Wave /14

If h(t) is a Heaviside step, the value of the pressure signature is given by:

$$\boxed{\frac{p_e(t)}{p_{e\max}} = \frac{1}{\tau_1}\int_0^t \left[h(t)-h(t-\tau_1)\right]dt + \frac{1+\lambda}{T-\tau_1-\tau_2}\int_0^t \left[h(t-T+\tau_2)-h(\tau_1)\right]dt + \frac{1}{\tau_2}\int_0^t \left[h(t-T+\tau_2)-h(t-T)\right]dt}$$

and its Laplace transform is:

$$\boxed{\mathcal{L}\left(\frac{p_e(t)}{p_{e\max}}\right) \doteq \frac{P_e(p)}{p_{e\max}} = \frac{1}{p^2}\left\{\frac{1}{\tau_1}\left(1-e^{-\tau_1 p}\right) + \frac{1+\lambda}{T-\tau_1-\tau_2}\left(e^{-(T-\tau_2)p} - e^{-\tau_1 p}\right) + \frac{1}{\tau_2}\left(e^{-(T-\tau_2)p} - e^{-Tp}\right)\right.}$$

1.3.2 Response of the Room to a N Wave Having Normal Incidence

We have seen that the room with an opening behaves like a resonator having the following characteristics:

ω_0 - eigenfrequency

S_0 - logarithmic decrement.

By setting $\boxed{r_0 = -\frac{S_0\,\omega_0}{2\pi}}$, the Laplace transform of the internal pressure $p_i(t)$ is:

$$P_i(p) = \frac{\omega_0^2 \, P_e(p)}{(p-r_0)^2 + \omega_0^2}$$

The decomposition into rational fractions gives:

$$\frac{1}{p^2\left[(p-r_0)^2 + \omega_0^2\right]}$$

in the form:

$$\frac{\alpha p + \beta}{p^2} + \frac{\gamma + j\delta}{p - r_0 - j\omega_0} + \frac{\gamma - j\delta}{p - r_0 + j\omega_0}$$

After transformation to the original plane of the Laplace transformation: /15

$$\begin{aligned} p_i(t) = {} & \frac{\omega_0^2}{r_0^2+\omega_0^2}\, p_e(t) + \frac{2 r_0 \omega_0^2}{r_0^2+\omega_0^2}\, \frac{dp_e}{dt} \\ & + \frac{A(r_0^2-\omega_0^2)\omega_0 - 2 r_0 \omega_0^2 B}{(r_0^2+\omega_0^2)^2}\, e^{r_0 t} \sin\omega_0 t \\ & + \frac{-2 A r_0 \omega_0^2 - B(r_0^2-\omega_0^2)\omega_0}{(r_0^2+\omega_0^2)^2}\, e^{r_0 t} \cos\omega_0 t \end{aligned} \tag{5}$$

A and B are given by:

$$\begin{aligned} \frac{A(t)}{p_{emax}} = {} & \frac{1}{\tau_1} - \left(\frac{1+\lambda}{T-\tau_1-\tau_2} + \frac{1}{\tau_1}\right) h(t-\tau_1) e^{-r_0 \tau_1} \cos\omega_0 \tau_1 \\ & + \left(\frac{1+\lambda}{T-\tau_1-\tau_2} + \frac{1}{\tau_2}\right) h(t-T+\tau_2) e^{-r_0 (T-\tau_2)} \cos\omega_0 (T-\tau_2) \\ & - \frac{h(t-T)}{\tau_2} e^{-r_0 T} \cos\omega_0 T \end{aligned}$$

$$\frac{B(t)}{p_{e\max}} = -\left(\frac{1+\lambda}{T-\tau_1-\tau_2} + \frac{1}{\tau_1}\right) h(t-\tau_1)\, e^{-r_0\tau_1} \sin\omega_0\tau_1$$
$$+ \left(\frac{1+\lambda}{T-\tau_1-\tau_2} + \frac{1}{\tau_2}\right) h(t-T+\tau_2)\, e^{-r_0(T-\tau_2)} \sin\omega_0 (T-\tau_2)$$
$$- \frac{h(t-T)}{\tau_2}\, e^{-r_0 T} \sin\omega_0 T$$

After an initial period equal to the total duration of the sonic boom, the pressure signature at the interior of the room is a damped sinusoid. During this initial period, the signature obtained is made up of several oscillations. According to the hypotheses which have been made, there are no longer any rapid increases of the overpressure (see Figures 1.7, 1.8, 1.9 and 1.10).

1.3.3 Response of the Resonator to a N Wave Having a Non-zero Incidence /16

The problem is to evaluate $\int_S p_e\, dS$. If the façade containing the opening is large compared with the wavelength under consideration (infinite baffle), we can write (see Figure 1.3)

$$p_e(x,t) = p_e\left(0, t - \frac{x \sin i}{c}\right)$$

$$\# \; p_e(0,t) - \frac{x \sin i}{c} \frac{dp_e}{dt}(0,t)$$

and:

$$\int_S p_e\, dS = p_e(0,t)S - \frac{L^2\ell}{2} \frac{\sin i}{c} \frac{dp_e}{dt}(0,t)$$

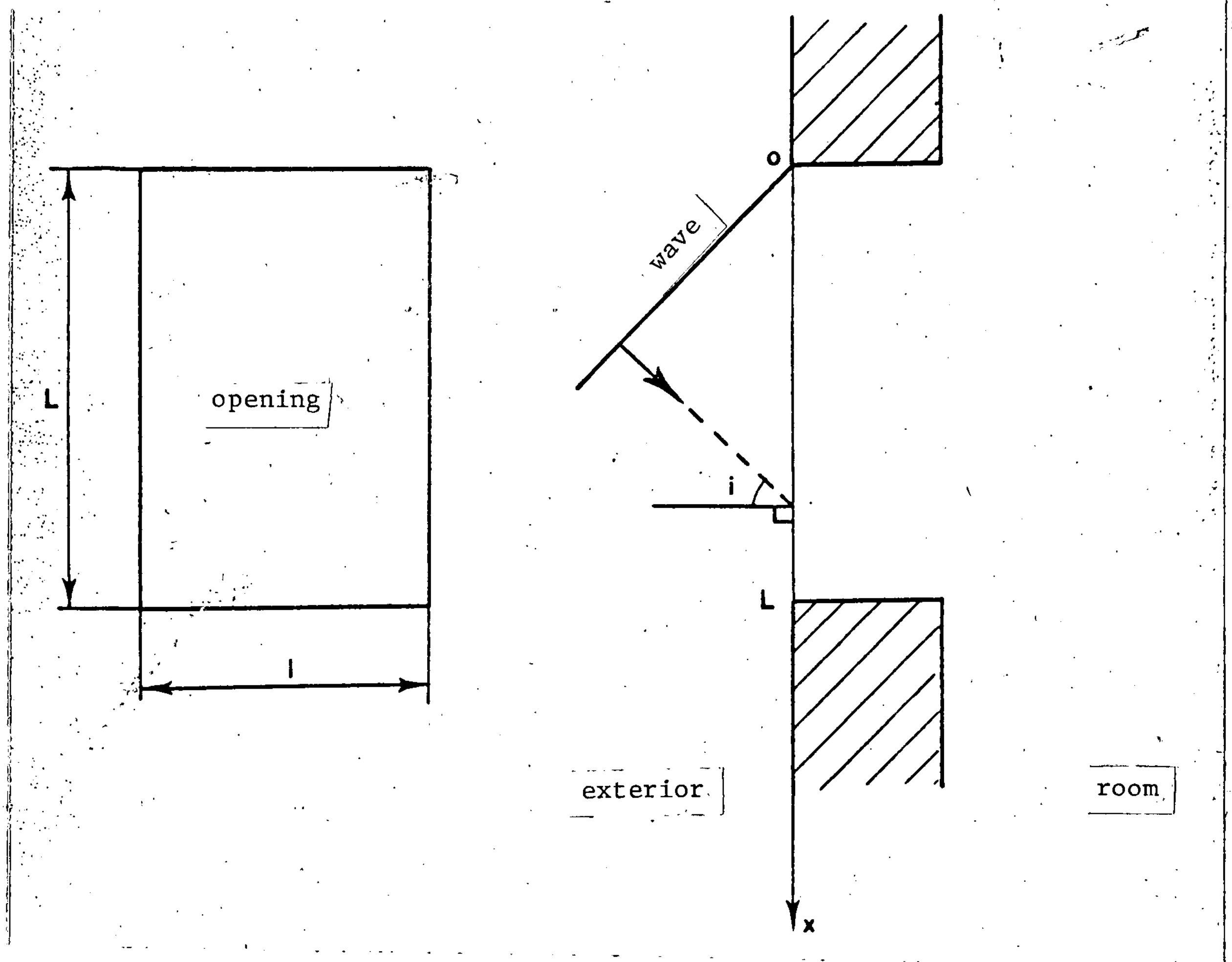


Figure 1.3. Effect of the angle of incidence i on the penetration of the sonic boom.

Assuming that $p_e(0,t)$ is known, Equation (5) results in a value $p_{io}(t)$, and we obtain:

$$p_i\ (t) = p_{io}\ (t) - \frac{L}{2}\ \frac{\sin i}{C}\ \frac{dp_{io}\ (t)}{dt}$$

Given the order of magnitude of the introduced coefficients, we will almost always have:

$$p_i\ (t) \# p_{io}\ (t)$$

This makes it possible to assume that we can write the following over the opening, assuming the façade of an arbitrary building which has an opening:

$$p_e(x, t) \neq p_e(o, t)$$

It is assumed that the opening is not very large (case of windows, doors).

1.4 INFLUENCE OF THE ELASTICITY OF THE ROOM WALLS /17

The study carried out previously for the penetration of a sonic boom through an opening in a room assumes that the walls of this room are rigid. The presence of a partition or a light ceiling can lead to substantial coupling between the modes of this panel and the modes of the room. The eigenfrequencies of the room and of the panel are modified and the overpressure is obtained by the sum of the responses to each of these modes. We will study the variation of the eigenfrequencies and the dampings, in order to determine the importance of the influence of the wall elasticity.

a) Equation for the internal pressure

If one of the walls of the room vibrates under the effect of the modified internal pressure of the room, and if the field in the room can still be assumed to be uniform, the overpressure in the room is given by (see Figure 1.4)

$$\frac{dp_i}{dt} = \frac{\rho c^2}{V}\int_S u_2\, dS - \frac{\rho c^2}{V}\sum_{\substack{m+n\\ \text{odd}}} \frac{4ab}{\pi^2 mn}\,\dot{q}_{mn}(t) \qquad (6)$$

This is valid for a panel having the dimensions a and b and simply supported. q_{mn} is the generalized displacement (see § 2.1). The summation of 1 up to infinity does not make sense. The wavelength of the eigenfrequencies of the panel becomes too small in order to provide a uniform field when m and n

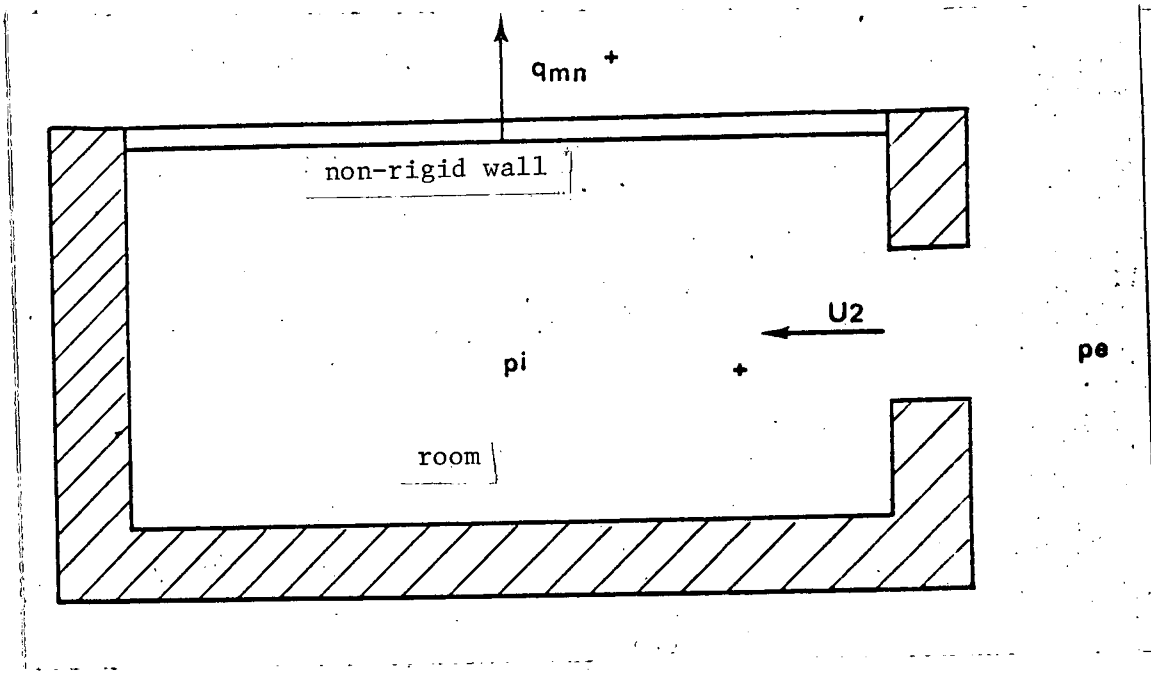


Figure 1.4. Influence of the elasticity of a wall of the room on the internal overpressure.

increase. However, the contribution of the higher frequency modes is negligible in general, considering the duration of the sonic boom and usual values for a and b.

In addition to Equation (6), we have the equilibrium equation for the resonator, in the form:

$$\int_S p_e\, dS = \alpha \int_S \frac{du_2}{dt}\, dS - \beta \int_S \frac{d^2 u_2}{dt^2}\, dS + \gamma \frac{d^2 p_i}{dt^2} - \delta \frac{d^3 p_i}{dt^3} + p_i S$$

from which, by replacing $\int_S u_2\, dS$ by its value obtained from (6):

$$\frac{1}{S}\int_S p_e\, dS = p_i + \frac{1}{\omega_0^2}\frac{d^2 p_i}{dt^2} - \frac{\delta_0}{\pi \omega_0^3}\frac{d^3 p_i}{dt^3} + \frac{4ab}{S\pi^2}\alpha \sum_{m+n\ \text{odd}} \frac{\ddot{q}_{mn}(t)}{mn} - \frac{4ab}{S\pi^2} \sum_{m+n\ \text{odd}} \frac{\dddot{q}_{mn}(t)}{mn}$$

../..

ω_0^* and δ_0^* are the circular frequency and logarithmic decrement of the system. /18

The walls of the room are assumed to be rigid.

q_{mn} is given by the Duhamel integral (zero initial conditions):

$$q_{mn}(t) = \frac{16}{\pi^2 mn \rho_p h \omega_{mn}} \int_0^t p_t(\tau)\, e^{-\beta_p \omega_{mn}(t-\tau)} \sin \omega_{mn}(t-\tau)\, d\tau$$

for a simply supported plate. The circular frequency of the mode (m, n) equals equals ω_{mn}. The damping is β_p, the thickness h, the volume mass is ρ_p. It is subjected to the total pressure $p_t(t)$.

We assume that the plate is only subjected to the pressure p_i, which will give an order of magnitude of its influence.

If $p_t = p_i$, the differential equation for p_i is written as:

$$\frac{1}{S}\int_S p_e\, dS = p_i + \frac{1}{\omega_o^2}\frac{d^2 p_i}{dt^2} - \frac{\delta_o}{\pi\omega_o}\frac{d^3 p_i}{dt^3} + \frac{64\, ab}{S\pi^4 \rho_p h} \sum_{m+n\ \text{odd}} \int_0^t \frac{\left(\alpha \frac{d^2 p_i}{dt^2} - \beta \frac{d^3 p_i}{dt^3}\right)(\tau)}{m^2 n^2 \omega_{mn}}\, e^{-\beta_p \omega_{mn}(t-\tau)} \sin \omega_{mn}(t-\tau)\, d\tau$$

For a sinusoidal pulsation wave ω, the second term can be written as :

$$p_i\left[1 - \frac{\omega^2}{\omega_o^2} + j\frac{\delta_o \omega^3}{\pi\omega_o^3} + \frac{64\, ab}{\rho_p h \pi^4 S} \sum_{m+n\ \text{odd}} \frac{-\alpha\omega^2 + j\omega^3\beta}{m^2 n^2\left[(j\omega + \beta_p\omega_{mn})^2 + \omega_{mn}^2\right]}\right]$$

b) <u>Determination of the Eigenfrequencies</u>

Taking the real part of expression (7), and setting it equal to zero, we obtain an equation in ω which is the equation for the eigenoscillations.

*Translator's Note: ω, δ omitted from foreign text.

Retaining only terms for m = n = 1, the system has only 2° of freedom, and the two eigenoscillations are the solution of the equation for ω_r:

$$\mathcal{R}_e = 1 - \frac{\omega_r^2}{\omega_0^2} + \frac{64ab}{\rho_p h \pi^4 S} \frac{\alpha\omega^8(\omega^2 - \omega_{11}^2) + 2\beta\omega^4\beta_p\omega_{11}}{(\omega_{11}^2 - \omega^2)^2 + 4\omega^2\beta_p^2\omega_{11}^2} = 0$$

In general, the terms containing β_p can be ignored if the solution obtained is not very close to ω_{11}. For this hypothesis, we obtain:

$$\mathcal{R}_e \# 1 - \frac{\omega_r^2}{\omega_0^2} + \frac{64\,ab}{\rho_p h \pi^4 S} \frac{\alpha\omega_r^8}{\omega_r^2 - \omega_{11}^2}$$

Since α is only slightly different from:

$\frac{\rho c^2 S}{\omega_0^2 V}$, by setting $\eta = \frac{64\,ab\rho c^2}{\rho_p h \pi^4 V \omega_{11}^2}$

(see § 2.2), we obtain:

$$1 - \frac{\omega_r^2}{\omega_0^2} + \eta \frac{\omega_{11}^2}{\omega_0^2(\omega_r^2 - \omega_{11}^2)} = 0$$

or

$$(\omega_r^2 - \omega_{11}^2)(\omega_0^2 - \omega_r^2) + \eta\omega_{11}^2 = 0$$

η is very small for a panel or a light ceiling (order of 1%). Therefore we find the two solutions:

$$\omega_{r1} \# \omega_{11}$$
$$\omega_{r2} \# \omega_0$$

This result shows that if $|\omega_0 - \omega_{11}|$ is large with respect to $\beta_p\omega_{11}$ (β_p is of order 1.01), which is the case as soon as ω_0 is only slightly different from ω_{11}, the eigenoscillation ω_0 is essentially one of the eigenoscillations of the system.

It should be noted that the result $\omega_{r1} = \omega_{11}$ is not valid, because it violates the hypothesis from which the analysis was started. From the point of view of the acoustic pressure, this is not important, because the pressure radiated by the wall must be negligible, and this problem will be treated in § 2.3.

Finally, if ω_0 is only slightly different from ω_{11}, then ω_r cannot be very different from ω_{11}, because the previous calculation would result in a contradiction, and ω_r is therefore a solution of

$$1 - \omega_r^2 \left(\frac{1}{\omega_0^2} - \frac{64\, ab\, \beta}{2 \rho_p h \pi^4 S \beta_p \omega_{11}} \right) = 0$$

or, essentially: /20

$$1 - \frac{\omega_r^2}{\omega_0^2} = 0$$

Thus we also obtain a solution $\omega_r \neq \omega_0$, and the overpressure will respond only to the pulsation mode ω_0.

c) Determination of the Damping for the Pulsation Mode ω_0

We can determine whether the damping β'_0 for the mode ω_0 is changed by the presence of a light panel. It is equal to the half of the imaginary part of expression (7), in which we set $\omega = \omega_0$. We obtain:

$$\beta'_0 = \frac{S_0}{2\pi} + \frac{64\, ab}{2 \rho_p h \pi^4 S} \sum_{m+n \text{ odd}} \frac{\beta \omega_0^3 (\omega_{mn}^2 - \omega_0^2) + 2\alpha \beta_p \omega_{mn} \omega_0^3}{m^2 n^2 \left[(\omega_{mn}^2 - \omega_0^2)^2 + 4 \omega_0^2 \beta_p^2 \omega_{mn}^2 \right]}$$

For $\omega_{mn} \neq \omega_0$, the terms containing β_p can be ignored. It follows that:

$$\beta'_0 = \frac{\delta_0}{2\pi} + \frac{\eta\,\delta_0\,\omega_{11}^2}{2\pi(\omega_{11}^2-\omega_0^2)} + \frac{\eta\,\delta_0\,\omega_{13}^2}{18\pi(\omega_{13}^2-\omega_0^2)} + \dots$$

which is almost equal to $\delta_0/2\pi$ for conventional values of η (order of 10^{-2}).

If the pulsation ω_{mn} is extremely close to ω_0, the imaginary part is essentially given by:

$$\beta'_0 = \frac{\delta_0}{2\pi} + \frac{\eta}{4\,m^2 n^2 \beta_p}$$

and the damping can be considerably increased. This is particularly true, the less damped the wall is and the lower the circular frequency ω_{mn} (m and n small).

d) Conclusion /21

The system consisting of the room with an opening and a simply supported and non-rigid wall must in general be considered to consist of the two subsystems coupled in a very loose way. The fact that the pressure is independent of the passage of the sonic boom through the opening and is independent of the wall vibration results in a small pressure radiated by this wall compared with the internal exciting pressure. This makes it possible to assume that:

1) the results will be identical for a wall having boundary conditions which are different from simple supports;

2) this wall is also excited along the outer wall of the room by the sonic boom. We may also assume that there may be a very loose coupling.

The case where an even lighter panel (window) is installed in the walls will be treated in section 2.3.

1.5 VERIFICATIONS OF THE REDUCED MODEL

a) Experimental Configuration

The linear acoustic equations (viscosity does not have an effect) show that it is possible to study the penetration of a sonic boom in a room having an opening, by using a model at the scale 1/n subjected to a ballistic detonation. The signature interval is divided by 1/n. The frequencies obtained will be n times the real frequencies [6].

The results obtained with models at higher frequencies should be transferrable to buildings having low frequencies.

Simulation of the Sonic Boom

The N wave was simulated by the explosion of goldbeaters skin balloons which were inflated by compressed air. If these balloons are new, the explosion will be quite uniform. The recording of the perturbation shows a wave whose form is quite close to that of a N.

The diameter of the various balloons vary between 85 cm and 140 cm at the moment of explosion. The signature interval obtained was within the range of 1.5 to 3.5 ms. The overpressure of the crest 3 meters from the balloon varied considerably (between 1 and 100 Pa in the free field). /22

This simulated N wave is in fact not perfect, because the rise times are relatively long (sometimes one quarter of the total duration). On the other hand, these rises are often not linear.

Model of the Room

A room having the internal dimensions 8 cm x 16 cm x 16 cm was built out of thick aluminum plates which were welded together. The façade consisted of a plate 5 mm in thickness. It had a variable and rectangular opening. This façade was placed in a baffle, above which the exploding balloons were mounted at a height of about 3 meters. Holes were provided for placing one quarter inch microphones (see Figure 1.5) and were located in the center of the three nonsymmetrical sides of the room.

Each side had an additional hole 2 cm from the level of the façade.

A microphone was installed in the baffle in order to record the signature at the outside. It was placed sufficiently close to the model so that it gave the same overpressure as at the level of the opening. The opening was closed and sufficiently far so that the radiation from this opening was negligible.

Measurement Configuration /23

The signals recorded by the microphones were directed to an oscilloscope after amplification. It was a memory type oscilloscope or had a photographic apparatus. Thus, signatures of the perturbations are easily obtained. The microphones used were the Brüel and Kjaer type 4135.

b) Measurement Results

Uniformity of the Field in the Interior of the Room

The overpressure obtained at the interior of the room varies according to the different modes. The oscillations for the higher modes always have a very small amplitude compared with those of the first mode.

We found that, according to the first mode, the overpressure varies in phase with the placing of the microphones, beginning after a very short time interval. The amplitude is almost uniform, according to the first mode, and decreases slightly when the opening is approached. The largest

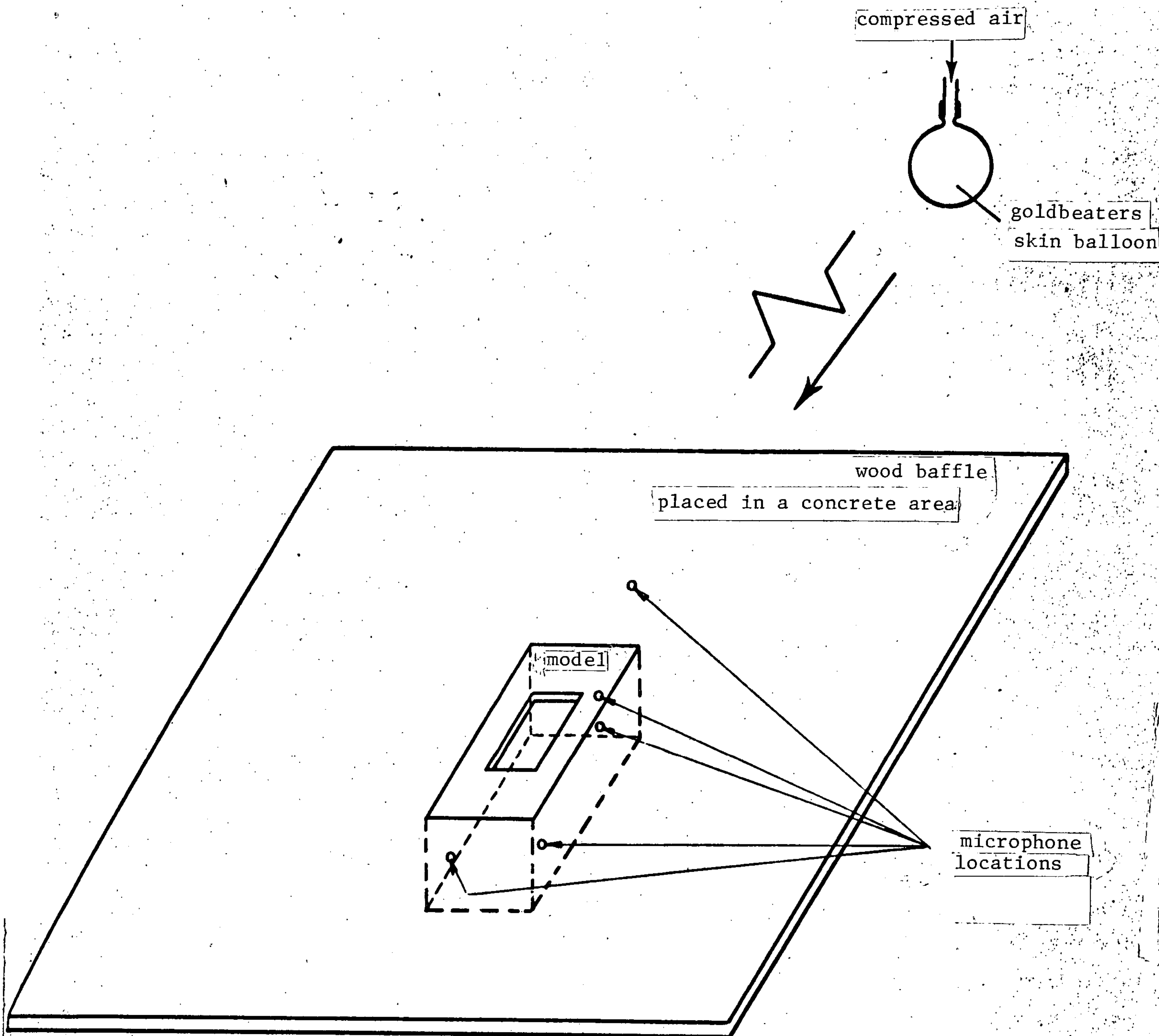


Fig. 1.5. Experimental configuration for the verification on models.

room dimension was equal to about 1/4 of the smallest wavelength observed in the experiments carried out.

In the center of the walls, for an incidence of 0° (incidence is the angle between the normal to the facade and the propagation direction), we found that the overpressure was the same at the center of the walls (except at the beginning) for all the modes. This result was not true for an incidence of 60°. The curves recorded by the photographic apparatus are shown in Figure 1.6.

Fundamental Eigenfrequency and damping

1) Measurements

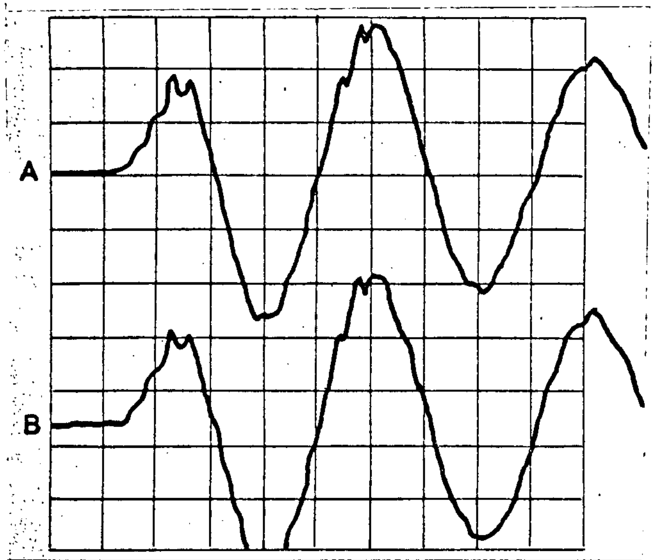
The rectangular opening had a constant length of 8 cm and the width had a value of 2.4 and 8 cm. This opening was placed in two positions:

- Position 1: the height of the façade (8 cm) is completely pierced.

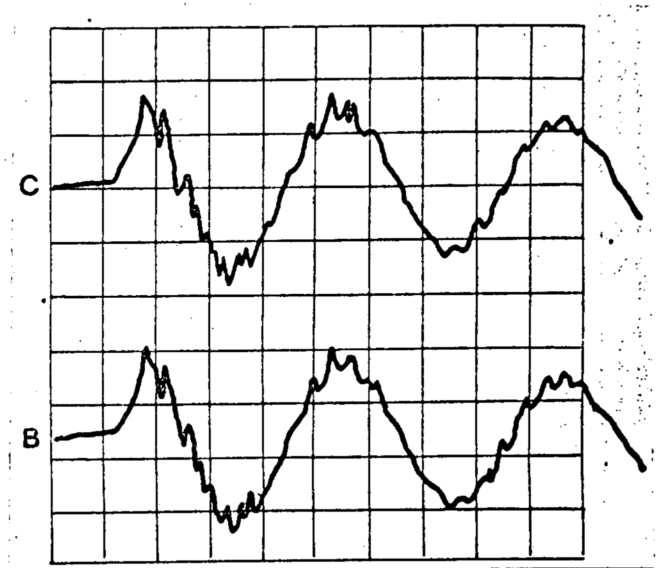
- Position 2: for widths equal to 2 and 4 cm, the opening is arranged in such a way that the length is parallel to the longest edge of the façade.

The measured values of the resonance frequencies and the logarithmic /24 damping are given in the table below. As a comparison, we also show the values calculated according to the two hypotheses of uniform pressure and velocity (see § 1.2).

The experimental determination of the resonance frequencies is quite accurate and can be estimated to be less than 5%. This is not true for the logarithmic decrement, and its uncertainty is assumed to be 15%. In practice, given the accuracy of the experiment, the logarithm of the ratio of two successive peaks is constant. We can assume that a more accurate measurement would show that the damping is not of a completely viscous nature.



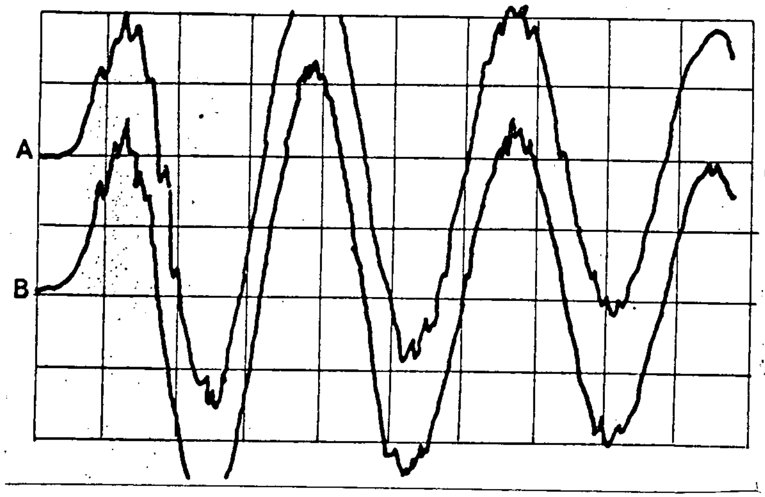
Overpressures recorded at locations A and B for the same incident waves i = 0° (dimensions 8 x 2 of the opening).



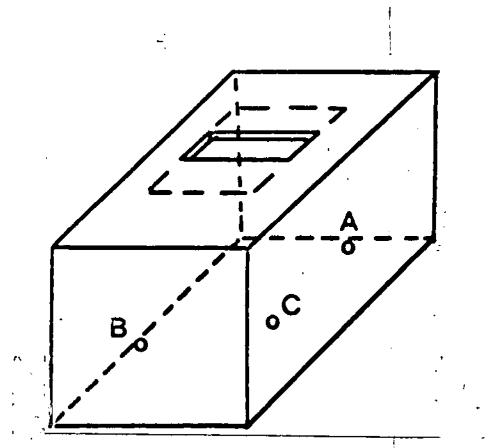
Overpressures recorded at locations B and C for the same incident wave i = 0° (dimensions 8 x 2 of the opening).

Figure 1.6. Uniformity of overpressures in the interior of the model.

(Figure continued on next page)



Overpressures recorded at locations A and B for the same incident wave i = 60° (dimensions 8 x 8 of the opening).



Locations A, B and C of the microphones

Figure 1.6 (continued)

ψ	Calculated values (uniform velocity)	Calculated values (uniform pressure)	Measured values (Position 1)	Measured values (Position 2)
1	f_R = 338 Hz δ = 0,50	f_R = 349 Hz δ = 0,55	f_R = 350 Hz δ = 0,29	
2	f_R = 282 Hz δ = 0,29	f_R = 291 Hz δ = 0,32	f_R = 290 Hz δ = 0,18	f_R = 310 Hz δ = 0,17
4	f_R = 241 Hz δ = 0,18	f_R = 246 Hz δ = 0,19	f_R = 245 Hz δ = 0,15	f_R = 260 Hz δ = 0,18

ϕ= ratio of the length to the width of the opening

f_R = frequency of resonance

δ = logarithmic decrement

We can estimate the experimental uncertainties in f_R and δ at 5% and 15%, respectively. The frequency calculation seems to be correct, but the damping obtained by this calculation is in general twice the true damping.

The latter result (ϕ = 4, positions 1 and 2) shows that it is important whether or not the walls are very close (this is not an isolated result, but was obtained several times). In any case, it is not the distance between the edges of the opening and the walls, expressed as a fraction of the wavelength of the resonant frequency, which is important, but undoubtedly the quotients of the length to the width of the opening and of the façade which matter. This conclusion is in agreement with the principles set forth in a study by Nesterov [7] concerned with a circular opening located in the center of a circular stream tube.

The experiments seem to show that when the walls approach each other:

$$\frac{f \text{ real}}{f \text{ theoretical}} \quad \text{diminishes}$$

and

$$\frac{\delta \text{ real}}{\delta \text{ theoretical}} \quad \text{also diminishes.}$$

In the case of an opening which occupies a large area in the façade, we could assume zero internal resistance to radiation, which would correspond with a model already given (see J. van Bladel, [1]).

The tests described above will be completed by tests on the model, which represents a building to be constructed at Istres. These tests will result in additional information.

Oscillations of the Acoustic Pressure

For an incident wave which has a shape very close to the letter N, the theoretical and experimental results agree very well when the true measured value of the damping is assumed in the calculations.

When the incident wave has numerous high frequency harmonics, the results will still agree, but the analogy with a N wave is a delicate one. The results of the calculations can change considerably for small variations in the models of the N wave.

It was not possible to measure the difference between the incidences i = 0° and i = 60°.

The response frequency of the resonator does not seem to depend on the amplitude of the incident wave, at least for the conditions in these experiments.

It was possible to observe a "dynamic amplification factor" of 2.5 to 3 for an incident wave which resembled a sinusoid more than a N wave, and which could not be treated by means of a simple mathematical model. It seemed that the resonance conditions were satisfied.

Theoretical and experimental curves are shown in Figures 1.7, 1.8, 1.9 and 1.10.

1.6 CONCLUSIONS

The study shows that the system consisting of a room and an opening can be considered as a Helmholtz resonator for the study of the penetration of a sonic boom.

Consequently, we can expect to obtain the pressure signature in the interior of a room which has the shape of a damped sinusoid. Its maximum will be equal to twice the overpressure of the incident sonic boom crest. This overpressure is measured on the façade and is more than two times the overpressure of the crest measured on the ground. This is true when the

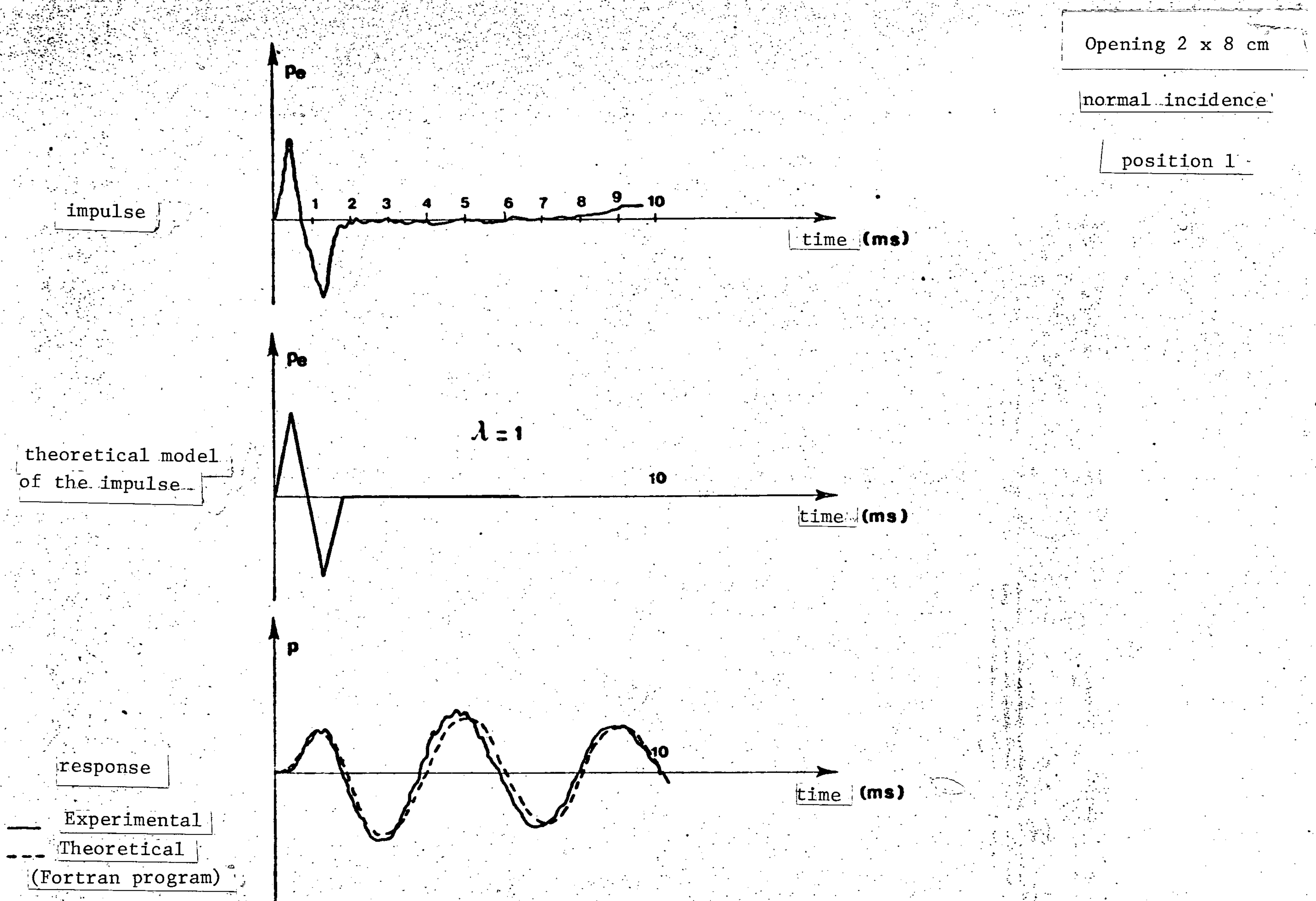


Fig. 1.7. Response of the models to a N wave.

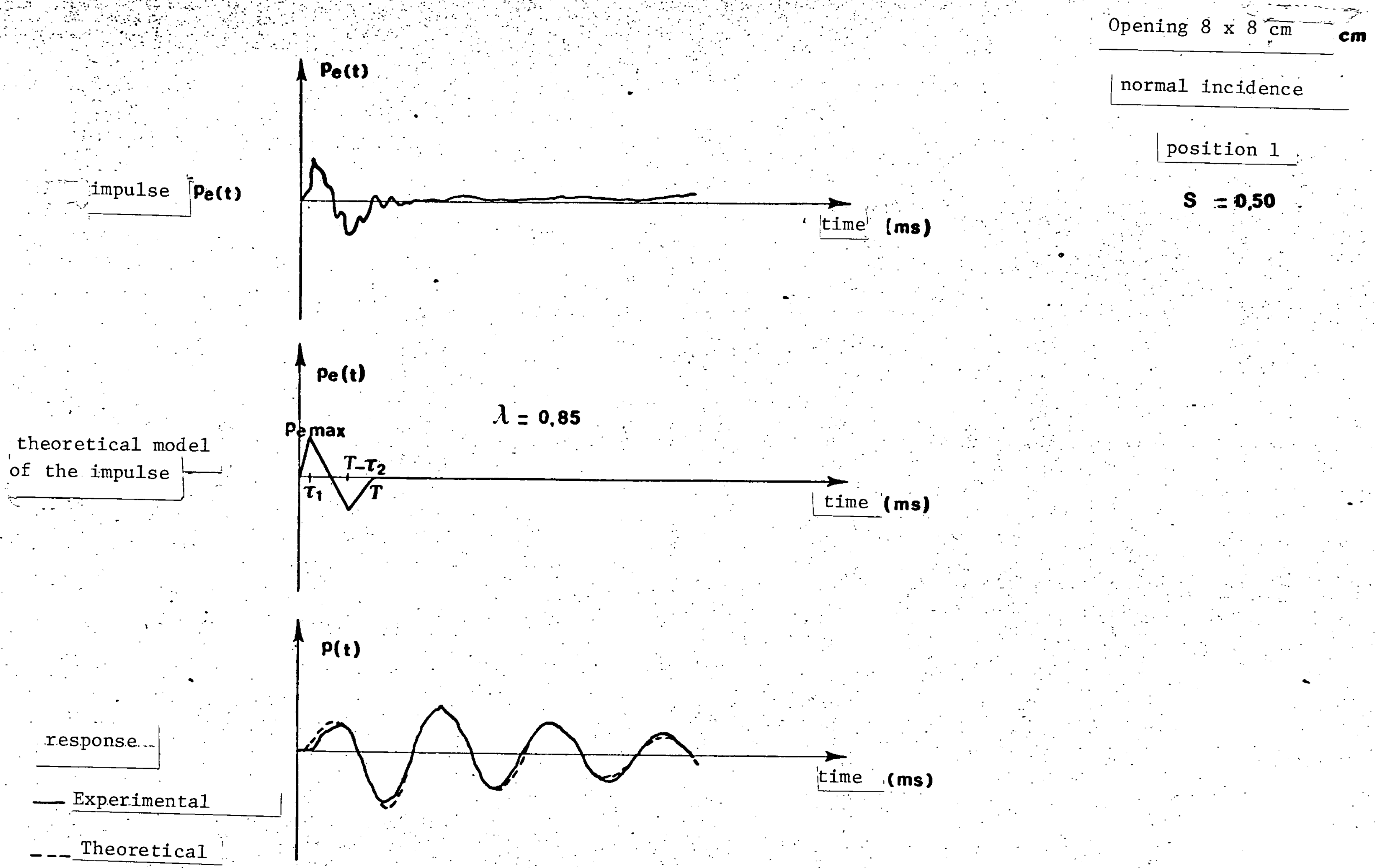


Fig. 1.8. Response of the models to a N wave.

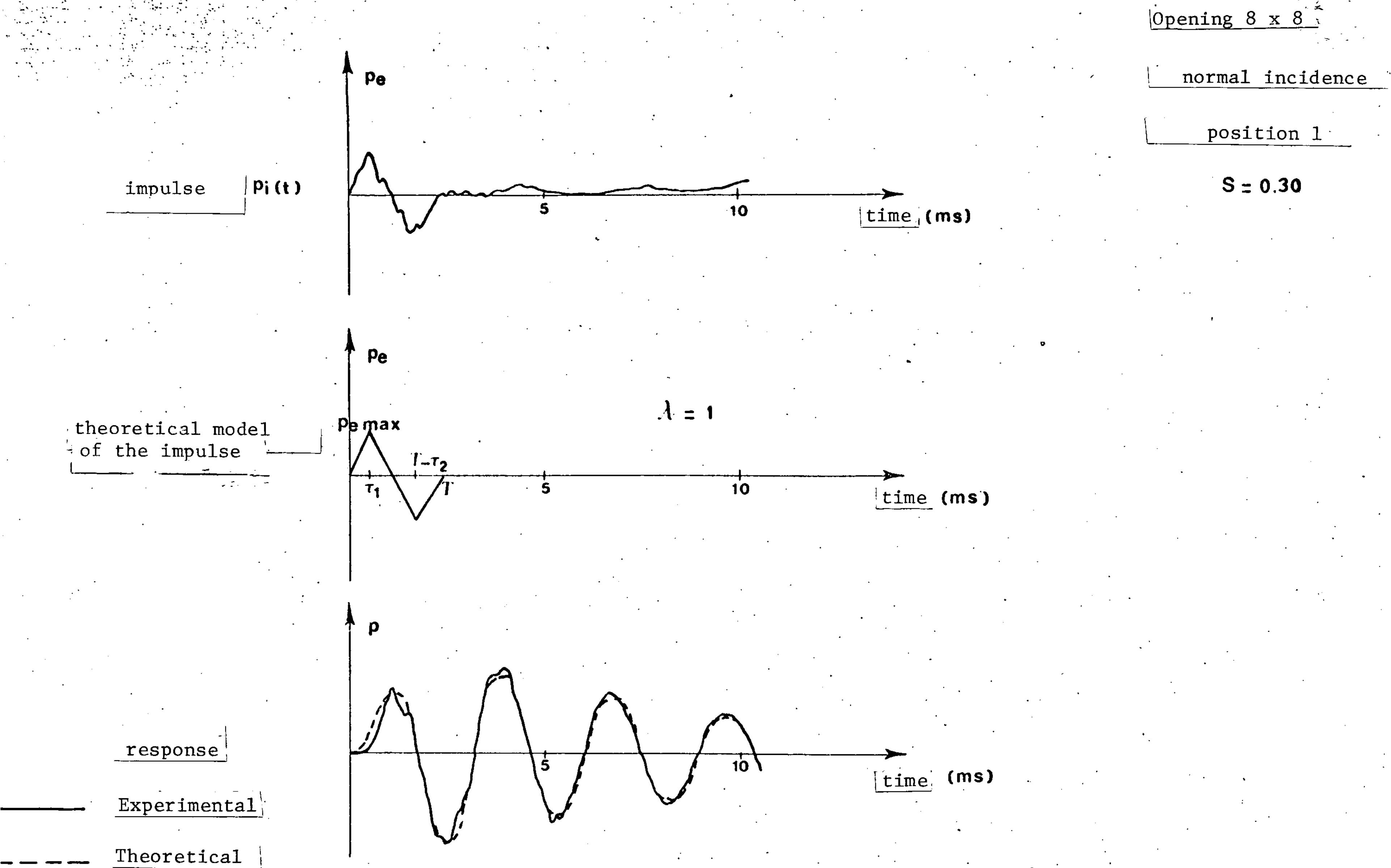


Fig. 1.9. Response of the models to a N wave.

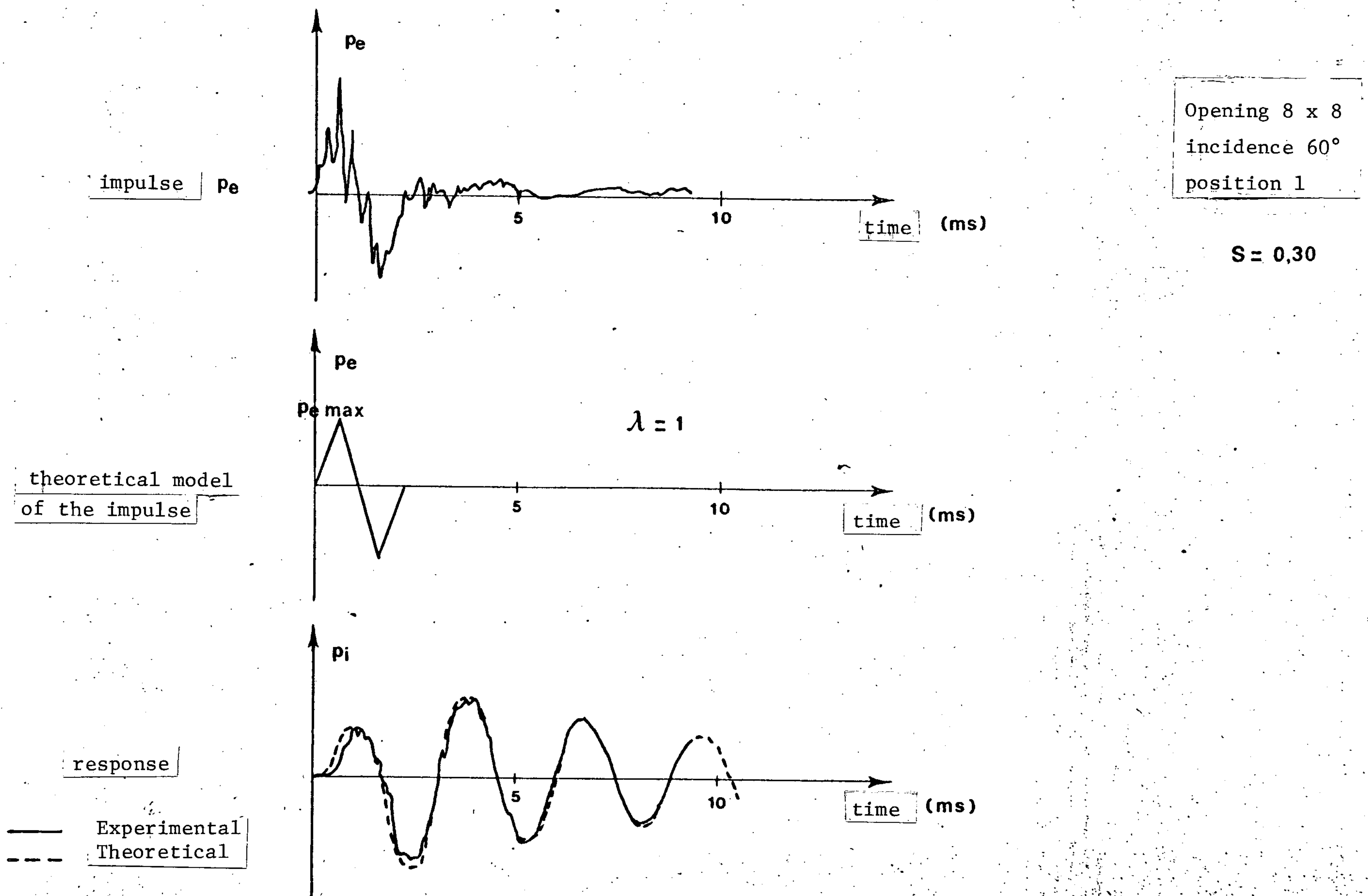


Fig. 1.10 Response of the models to a N wave.

signature interval of the sonic boom is approximately equal to the period corresponding to the eigenfrequency of the system consisting of the room and the opening.

The hypotheses made do not make it possible to predict the rise time (or the greatest slope) of the internal overpressure.

In order to obtain resonance with a classic supersonic aircraft (fighter), an eigenfrequency of about 10 Hz is required, which is usually found (see § 1.2). With a supersonic transport aircraft of the Concorde type, an eigenfrequency of 3 Hz is required, which is only obtained for a very small opening with respect to the room. On the other hand, a double resonator (two rooms connected by an open door, and the sonic boom penetrates into one of them through an opening) can have an eigenfrequency of this order. In addition, the overpressures obtained can be considerably higher. The study of the double resonator can be carried out in connection with the preceding study.

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2. VIBRATIONS OF WALLS IN A ROOM SUBJECT TO OVERPRESSURE

2.1 SUMMARY OF THE CLASSICAL THEORY OF VIBRATION OF A HOMOGENEOUS WALL /30

The following calculation treats the vibrations of a homogeneous rectangular plate, having length a, width b and thickness h. It is assumed that the membrane stresses can be ignored (which is assumed for a deflection smaller than the thickness). Also it is assumed that there are no internal prestresses. These simplifications make it possible to obtain the linear relationships given below.

a) General Equation

The general differential equation can be written as

$$\frac{\partial^2 w}{\partial t^2} + \frac{E h^2}{12 \rho_p (1-\nu^2)} \left(\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) = \frac{p(x,y,t)}{\rho_p h} \quad (1)$$

with:
- w (x, y, t) = dynamic deflection
- E = modulus of elasticity
- ν = Poisson coefficient
- ρ_p = volume mass
- p (x, y, t) = pressure applied to the wall.

In addition to this equation, boundary conditions at the edge of the plate must be specified.

If p (x, y, t) does not depend on w, which assumes negligible radiation, we can find eigenmodes in the form:

$$\varphi_r(x,y) \sin \omega_r t$$

which satisfy:

$$\frac{Eh^2}{12\rho_p(1-\nu^2)}\left(\frac{\partial^4\psi_r}{\partial x^4}+2\frac{\partial^4\psi_r}{\partial x^2\partial y^2}+\frac{\partial^4\psi_r}{\partial y^4}\right)=\omega_r^2\psi_r$$

and the boundary conditions at the edge of the wall.

If we make a decomposition according to the various eigenmodes, in such a way that: /31

$$w=\sum_{r=1}^{\infty}q_r(t)\,\psi_r(x,y)$$
$$p=\sum_{r=1}^{\infty}p_r(t)\,\psi_r(x,y)$$

we obtain for each mode:

$$\ddot{q}_r+\omega_r^2 q_r=\frac{1}{\rho_p h}p_r(t)$$

We can add a damping term β_p and we obtain:

$$\ddot{q}_r+2\beta_p\omega_r\dot{q}_r+\omega_r^2 q_r=\frac{1}{\rho_p h}p_r(t) \qquad (2)$$

$p_r(t)$ is given by:

$$p_r(t)=\frac{\int_s p(x,y,t)\,\psi_r(x,y)\,dx\,dy}{\int_s \psi_r^2(x,y)\,dx\,dy}$$

and $q_r(t)$ can be determined by a Duhamel integral:

$$q_r(t)=\int_0^t\frac{p_r(\tau)}{\rho_p h\omega_r}e^{-\beta_p\omega_r(t-\tau)}\sin\omega_r(t-\tau)\,d\tau \qquad (3)$$

In order to obtain the stresses we have the relationship:

$$\sigma_x = -\frac{Eh}{2(1-\nu^2)}\left[\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right]$$

$$\sigma_y = -\frac{Eh}{2(1-\nu^2)}\left[\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right]$$

b) <u>Case of a Simply Supported Wall</u>

The boundary conditions for a real wall are not well defined in general. In order to simplify the calculations, frequently the case of simple supports is considered[2].

In this case: /32

$$\varphi_{mn}(x,y) = \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

(a and b are the dimensions of the wall)

satisfy the imposed conditions. We obtain:

$$\omega_{mn} = \pi^2\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)\left(\frac{D}{\rho_p h}\right)^{\frac{1}{2}}$$

with

$$D = \frac{Eh^3}{12(1-\nu^2)}$$

$$p_{mn}(t) = \frac{4}{ab}\int_S p(x,y,t)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\,dx\,dy$$

[2] The case of clamping is formally identical. The frequencies and eigenmodes are given by M. J. Crocker, for example "Multimode Response of Panels to Normal and Traveling Sonic Booms", J.A.S.A., Vol. 42, No. 5, November, 1967.

and if p only depends on t:

$$\begin{cases} p_{mn}(t) = \dfrac{16}{\pi^2 mn} p(t) & \text{for } m+n \text{ odd} \\ p_{mn}(t) = 0 & \text{for } m+n \text{ even} \end{cases}$$

We obtain the following for the stresses:

$$\sigma_x = \frac{Eh\pi^2}{2(1-\nu^2)} \sum_{m+n \text{ odd}} \left(\frac{m^2}{a^2} + \nu \frac{n^2}{b^2}\right) \Psi_{mn}(x,y) q_{mn}(t)$$

$$\sigma_y = \frac{Eh\pi^2}{2(1-\nu^2)} \sum_{m+n \text{ odd}} \left(\frac{n^2}{b^2} + \nu \frac{m^2}{a^2}\right) \Psi_{mn}(x,y) q_{mn}(t)$$

(stresses are a maximum in the center).

For the acceleration we can assume:

$$w'' = -\sum_{m+n \text{ odd}} \omega_{mn}^2 q_{mn}(t) \Psi_{mn}(x,y)$$

c) Application to the Case Where the Pressure acting on the Wall is a Wave Shaped like a N /33

Numerous studies have been made on the subject. The displacements, stresses, accelerations are the sums of damped sinusoids (see Figure 2.1), which is confirmed by experiment.

It is expedient to introduce the notion of the dynamic amplification factor (D.A.F.) defined in general as the ratio of the maximum dynamic displacement due to the sonic boom and the displacement which would be obtained by a static load equal to the overpressure of the crest and uniformly distributed.

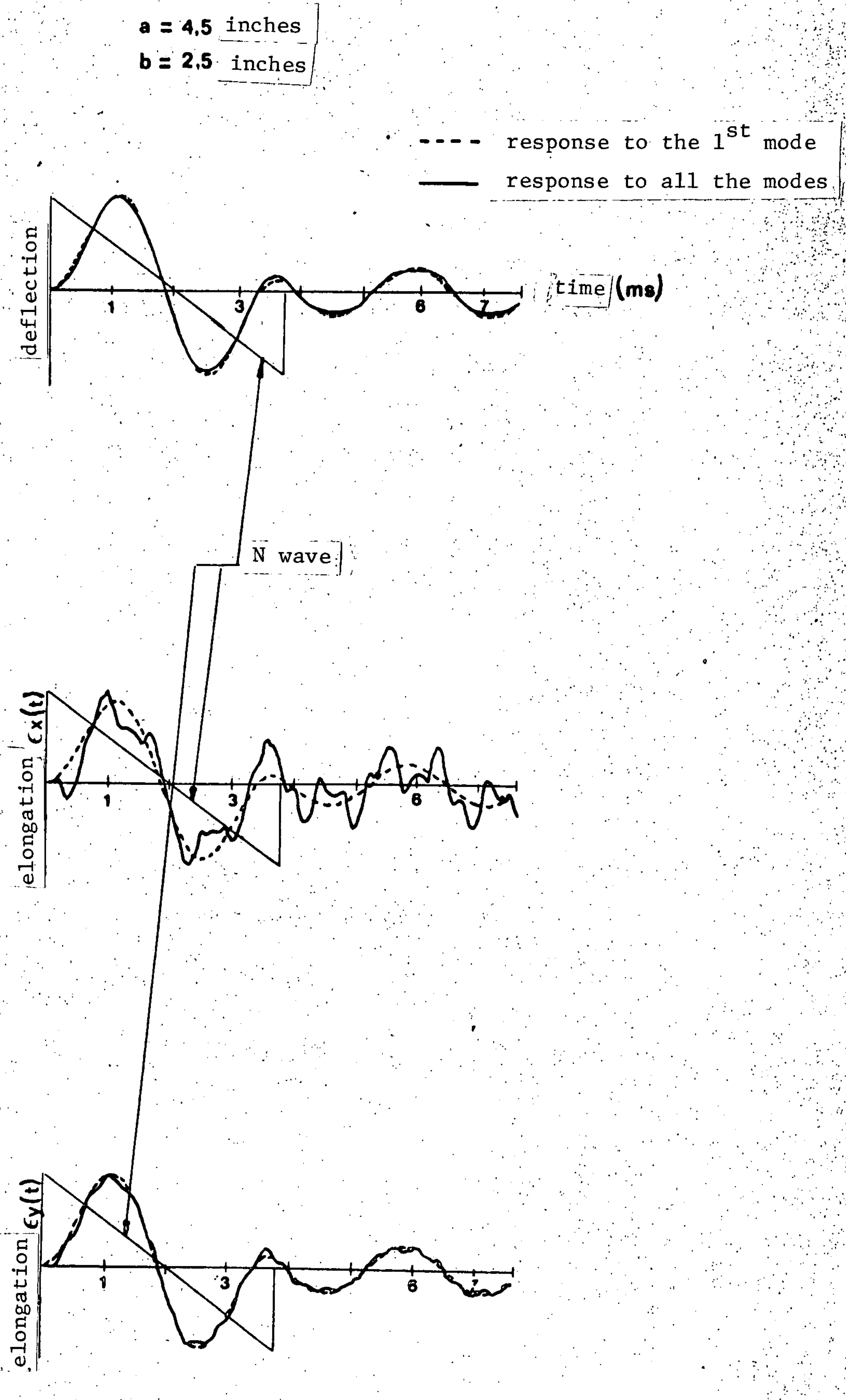

Figure 2.1. Displacement and stresses at the center of a simply supported plate (according to [7]) subjected to a normal N wave.

Numerous curves of D.A.F. have been drawn, (see Figure 2.2) for a reduced structural element and the first mode alone, as well as by taking into account all the modes.

The maximum dynamic amplification is on the order of 2.

2.2 CASE WHERE THE WALL IS LIMITED BY A CLOSED ROOM (ALONG ONE OF THE SIDES)

The second term of the general differential equation (1)

$$\rho_p h \frac{\partial^2 w}{\partial t^2} + \frac{E h^3}{12(1-\nu^2)} \left(\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) = p(x,y,t)$$

can be considered as being made up of two overpressures:

1) the incident overpressure p_e (x, y, t) /34

2) the overpressure radiated by the vibrations of the wall p_r (x, y, t), to the exterior as well as to the interior of the room (see Figure 2.4).

In a free field, this overpressure p_r (x, y, t) is negligible compared with the overpressure p_e (x, y, t). On the other hand, if there is a room adjacent to the plate, the quantity p_r (x, y, t) will increase because of the reflections on the walls of this room of the radiated wave. Consequently, it is appropriate to evaluate the influence of a room adjacent to the panel, because this situation always exists in practice.

We will expect the following:

1) there will be possibly a detectable modification of the vibration amplitudes of the plate

2) p_r (x, y, t) depends on the displacement w (x, y, t), for any change in the eigenmodes and the eigenfrequencies of the plate.

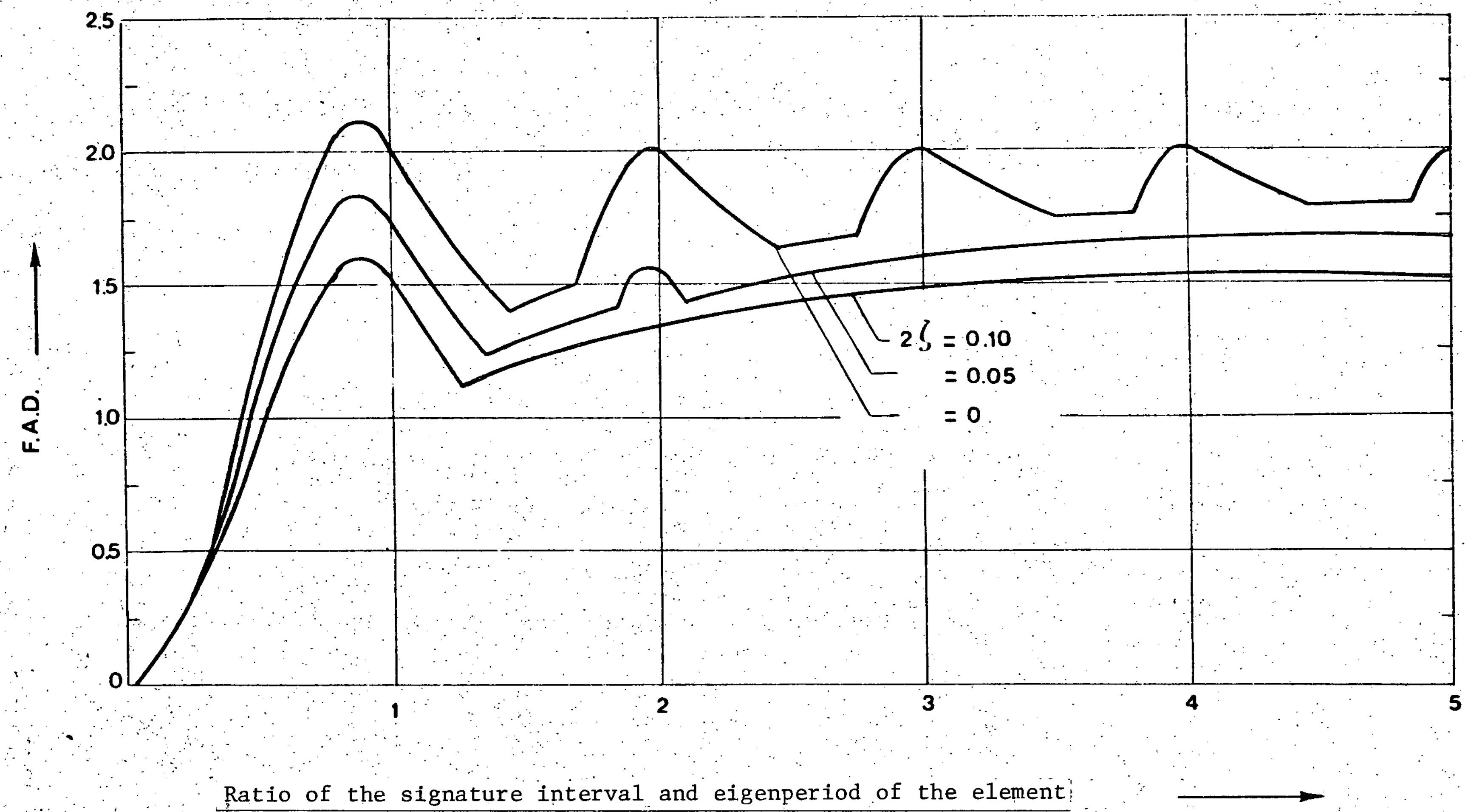

Figure 2.2. Dynamic amplification factor (D.A.F.) for a N wave (ζ = damping) (according to [8]).

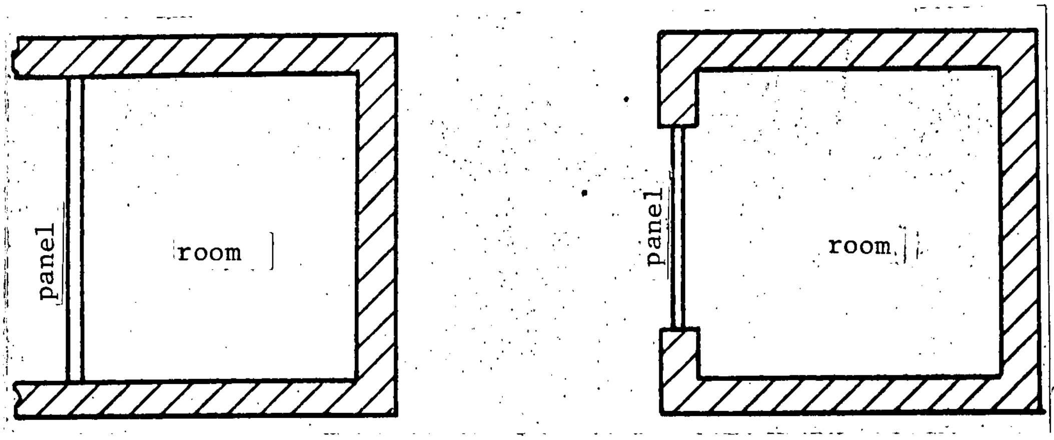

Figure 2.3. Case where the wall is bounded by a closed room.

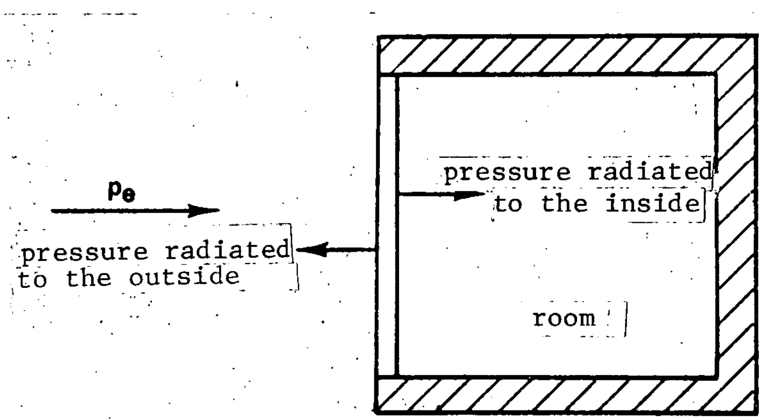

Figure 24. Radiation of the panel.

We will first deal with the case of a closed room, having rigid and non-absorbing walls adjacent to the plate, and we will discuss studies which have already been carried out.

a) Radiation of the Panel

The simplest hypothesis is to assume that the overpressure in the room is uniform. Craggs [1] considered one reasonable limit of this hypothesis, which is that the first in situ frequency of the panel under consideration is smaller than one half of the fundamental frequency f_0 of the room, which is considered as a tube open at one end (the one containing the panel) and closed at the other end. This in effect assumes that the panel covers all of one wall of the room. Assuming that the occupied area is less than one half of the surface of the wall of the room, the transverse modes of the room must contribute to the uniformity of the pressure field. Assumming that the first frequency of the panel is less than the frequency f_0, a finer analysis can be carried out assuming an expression of the following form for the radiated pressure:

$A(t)\cos\omega\frac{z}{c}$, for an incident sinusoidal wave with pulsation ω. The abscissa Z = 0 corresponds to the wall opposite the panel [1].

This hypothesis corresponds to a plane wave in the room. It assumes in particular that the panel occupies a large part of one of the walls of the room. For this configuration, Craggs [1] was able to theoretically confirm this hypothesis using the method of finite elements. He also found that, if primarily the first frequency of the panel is heard on the floor of the room, the higher frequencies will become more and more perceptible as one approaches the panel carrying out vibrations. /35

Besides the panel dimensions compared with the dimensions of the wall containing the panel, a second restriction to the planar wave model is that the depth d of the cavity cannot be too small compared with the largest dimension L of the panel. For d < L/2, the transverse modes will become more important [3].

Assuming uniformity, the interior overpressure has the following value, using the notation of §§ 1.1 and 2.1

$$p_i(t) = \frac{\rho c^2 a b}{V}\int_S w\,dS$$ (see Figure 2.5)

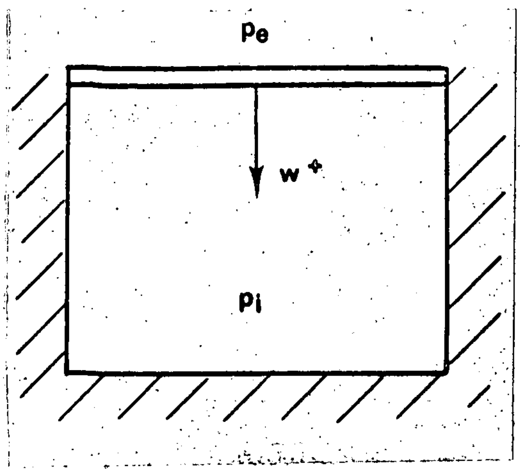

Figure 2.5. Diagram for the calculation of the interior overpressure.

and for a simply supported panel this results in:

$$p_i(t) = \frac{4\rho c^2 ab}{\pi^2 V} \sum_{m+n \text{ odd}} \frac{q_{mn}(t)}{mn}$$

The summation over all odd and whole m and n does not make sense, because for very high frequencies, the pressure can no longer be uniform. However, the duration of a ballistic detonation is such that it will primarily excite the (1, 1) mode, and the contribution of the other modes (m, n) to the radiated pressure become negligible when m and n are increased.

b) Vibration of a Simply Supported Panel Adjacent to the Room /36

If we make a decomposition according to the base vectors

$$\psi_{mn} = \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

the generalized displacement is given by Equation (3):

$$q_{mn}(t) = \frac{16}{\pi^2 \rho_p h \omega_{mn} mn}\int_0^t p_t(\tau)\, e^{-\beta_p \omega_{mn}(t-\tau)} \sin\omega_{mn}(t-\tau)\, d\tau$$

p_t is the total overpressure. We will ignore the pressure radiated by the plate into a free field, so that:

$$p_t = p_e - p_i$$

It should be noted that this radiated pressure in the free field can be included in β_p, which is measured and which contains not only the purely mechanical damping due to the structure of the plate and its support conditions.

Under these conditions, assuming uniform pressure:

$$q_{mn}(t) = \frac{16}{\pi^2 \rho_p h \omega_{mn} mn} \int_0^t p_e(\tau) e^{-\beta_p \omega_{mn}(t-\tau)} \sin \omega_{mn}(t-\tau) d\tau$$

$$- \frac{64 ab}{\rho_p h \pi^4 \omega_{mn} mn} \frac{\rho c^2}{V} \sum_{r+s \text{ odd}} \int_0^t \frac{q_{rs}(\tau)}{rs} e^{-\beta_p \omega_{mn}(t-\tau)} \sin \omega_{mn}(t-\tau) d\tau$$

In carrying out a Laplace transformation of the two terms, we obtain: (zero initial conditions)

$$\Phi_{mn}(p) = \frac{16}{\pi^2 \rho_p h\, mn} \frac{P_e(p)}{\left[(p+\beta_p \omega_{mn})^2 + \omega_{mn}^2\right]} - \frac{64\, ab\, \rho c^2}{\rho_p h \pi^4 V mn} \sum_{r+s \text{ odd}} \frac{\Phi_{rs}}{rs\left[(p+\beta_p \omega_{mn})^2 + \omega_{mn}^2\right]} \qquad (4)$$

In order to have an order of magnitude, we can limit ourselves to m = n = r = s = 1, and we obtain:

$$\left[(p+\beta_p \omega_{11})^2 + \omega_{11}^2 + \frac{64\, ab\, \rho c^2}{\rho_p h \pi^4 V}\right] \Phi_{11} = \frac{16}{\pi^2 \rho_p h} P_e(p)$$

This expression shows that the room behaves like an additional spring, and that the in situ frequency of the plate is increased. The ratio: /37

$$\eta = \frac{64\, ab\, \rho c^2}{\rho_p h \pi^4 V \omega_{11}^2} \quad \text{or} \quad \boxed{\eta = \frac{768(1-\nu^2)\rho c^2}{\pi^8 h^3 V E} \frac{a^5 b^5}{(a^2+b^2)^2}}$$

determines the importance of the room influence.

The first eigenfrequency of the plate is then given by:

$$F_{11}^2 = f_{11}^2 (1+\eta)$$

f_{11} is the eigenfrequency in the free field.

The importance of η makes it possible to predict the largest or the smallest influence of the closed room.

- For a conventional wall (made of plaster) making up one side of a room having the usual dimensions (4 x 4 x 2.5 m), η is negligible and there is hardly any coupling.

- For an average window (1.30 x 1.30), consisting of two cross pieces having the usual thicknesses (1.95 mm), in the same room as before, η is also negligible and there should be no coupling. It should be remarked that the first frequency of the window approaches the fundamental frequency fo of the room, and the hypothesis of uniform pressure will be subject to a certain error.

- For a large bay having glass panes with the dimensions 4 x 2.5 (thickness 6 mm) on a room having a depth of 4 meters, η is no longer negligible ($\eta = 4$).

The preceding calculation must be improved if η becomes too large. One method consists of summing m and r from 1 to 2R + 1 in the Equations (4), and summing n and s from 1 to 2S + 1. Then the obtained system of equations is solved. Pretlove and Craggs showed that a good approximation is found [5] if we restrict ourselves to the two first odd modes for a rectangular plate (but not for a square one). In particular, the shape of these modes is obtained with sufficient accuracy.

This calculation assumes that the hypothesis of uniform pressure is /38 satisfied, which cannot always be realized. The calculations of the radiated pressure, carried out up to the present, assume that the first frequency of the panels does not exceed the fundamental frequency fo of the room (which is usually the case). In the opposite case, the latter acts like a supplementary air mass [3] and [1].

Using the hypothesis of a plane wave in the interior of the room and for an incident sinusoidal wave with circular frequency ω, we then find the ratio η equal to [1]:

$$\frac{64\,\rho c^2 ab}{\pi^4 \rho_p h V}\,\frac{\omega d}{c}\,\cot g\,\frac{\omega d}{c}$$

which shows that the volume of the room behaves like a component with negative rigidity (which amounts to an additional mass).

Even though the plane wave hypothesis is not very realistic, a detailed study of the response of the system consisting of the panel and the cavity can be made using the method of finite elements [1]. The results obtained for the eigenfrequencies are shown in Figure 2.6.

The modification of the eigenmodes and the eigenfrequencies makes it possible to predict the response of the system. The amplitude of displacement of the plate is reduced, approximately by the ratio 1 + η if η is not too large. The amplitude of the stresses vary essentially like that of the displacements, except when η is too large [1]. The overpressure in the interior of the room increases with η, on the other hand.

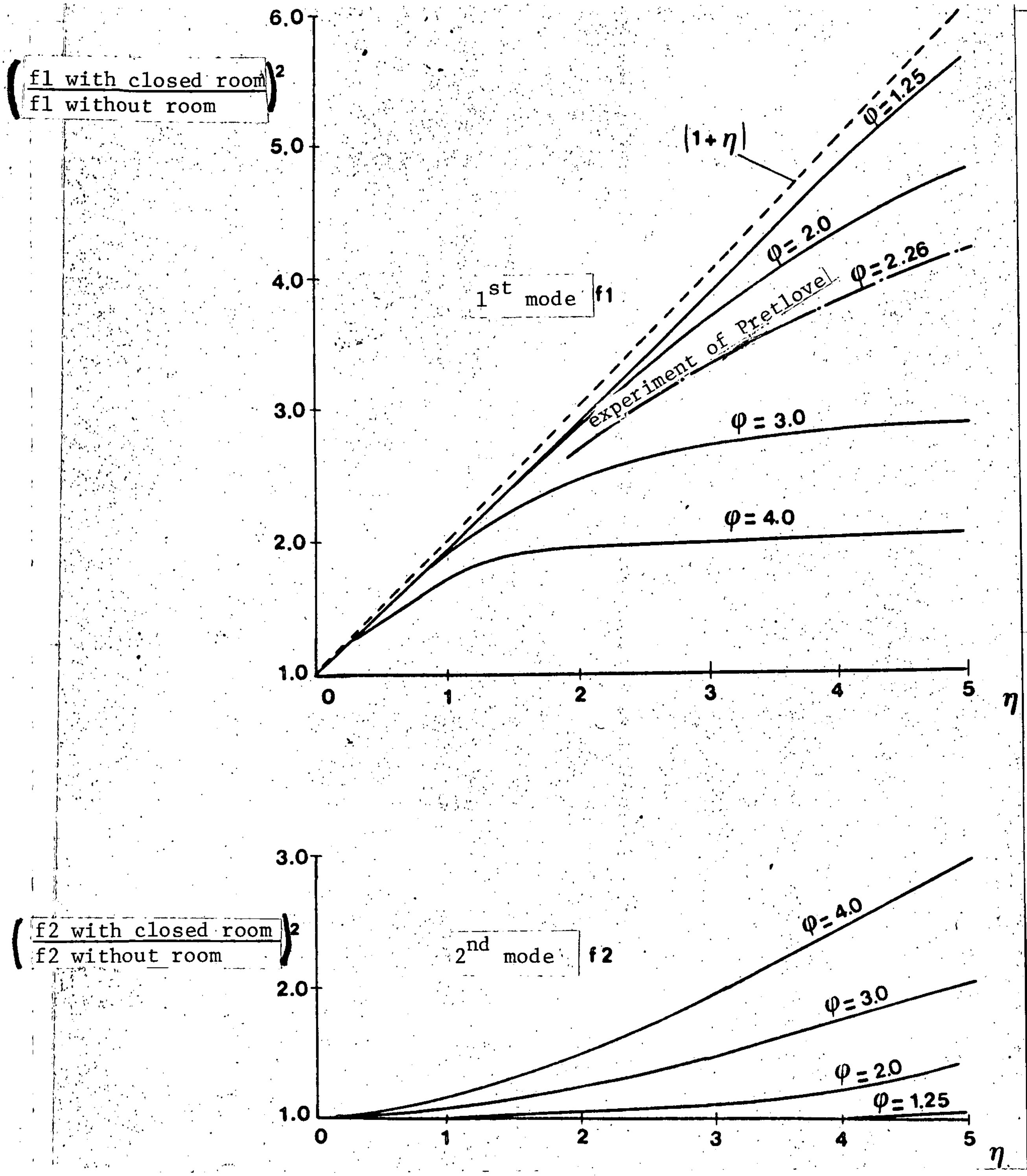

Figure 2.6. Variation of the eigenfrequencies of a simply supported plate with η and the ratio of length to width ϕ (according to [1]).

The previous calculations were carried out in the case of a simply supported plate. The case of a clamped plate can be treated in the same way. The coupling is smaller. The results of eigenfrequencies are shown in Figure 2.7 according to [1].

2.3 CASE WHERE THE WALL IS LIMITED BY AN OPEN ROOM /39

The panel can occupy a part of or the entire wall of a room having an opening (see Figure 2.8) and the response of this panel can depend greatly on this situation.

The closeness of the room brings about a coupling between the radiated pressure and the vibrations of the panel, just as in § 2.2.

This problem was treated in § 1.4 for a simply supported plate.

If the coupling becomes important, one cannot take the dampings into account, so that the equation for the eigenpulsations is written as (see § 1.4):

$$1-\frac{\omega^2}{\omega_0^2}-\sum_{m+n\ \text{odd}}\frac{64\,ab\,\alpha\,\omega^2}{\rho_p h\pi^4 S}\,\frac{1}{m^2n^2(\omega_{mn}^2-\omega^2)}=0$$

where, since $\alpha \# \dfrac{\rho c^2 S}{\omega_0^2 V}$:

$$1-\frac{\omega^2}{\omega_0^2}-\eta\frac{\omega_{11}^2}{\omega_0^2}\sum_{m+n\ \text{odd}}\frac{1}{m^2n^2(\omega_{mn}^2-\omega^2)}=0$$

η was defined in § 2.2.

a) <u>Solving for Eigenfrequencies</u>

If we only retain the first mode of the panel, the equation is written as:

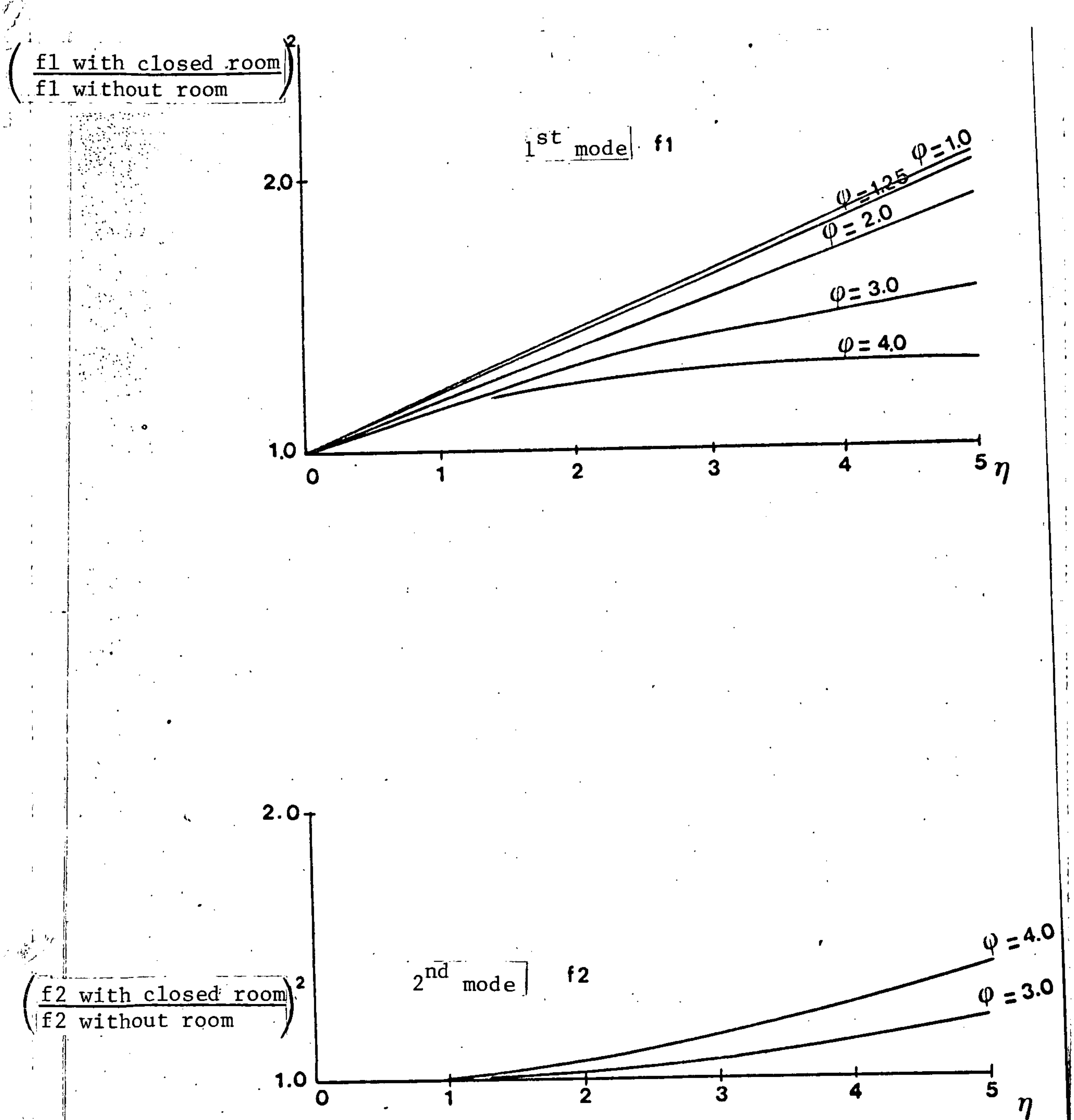

Figure 2.7. Variation of the eigenfrequencies of a simply-supported plate with η and the ratio of length to width ϕ (according to [1]).

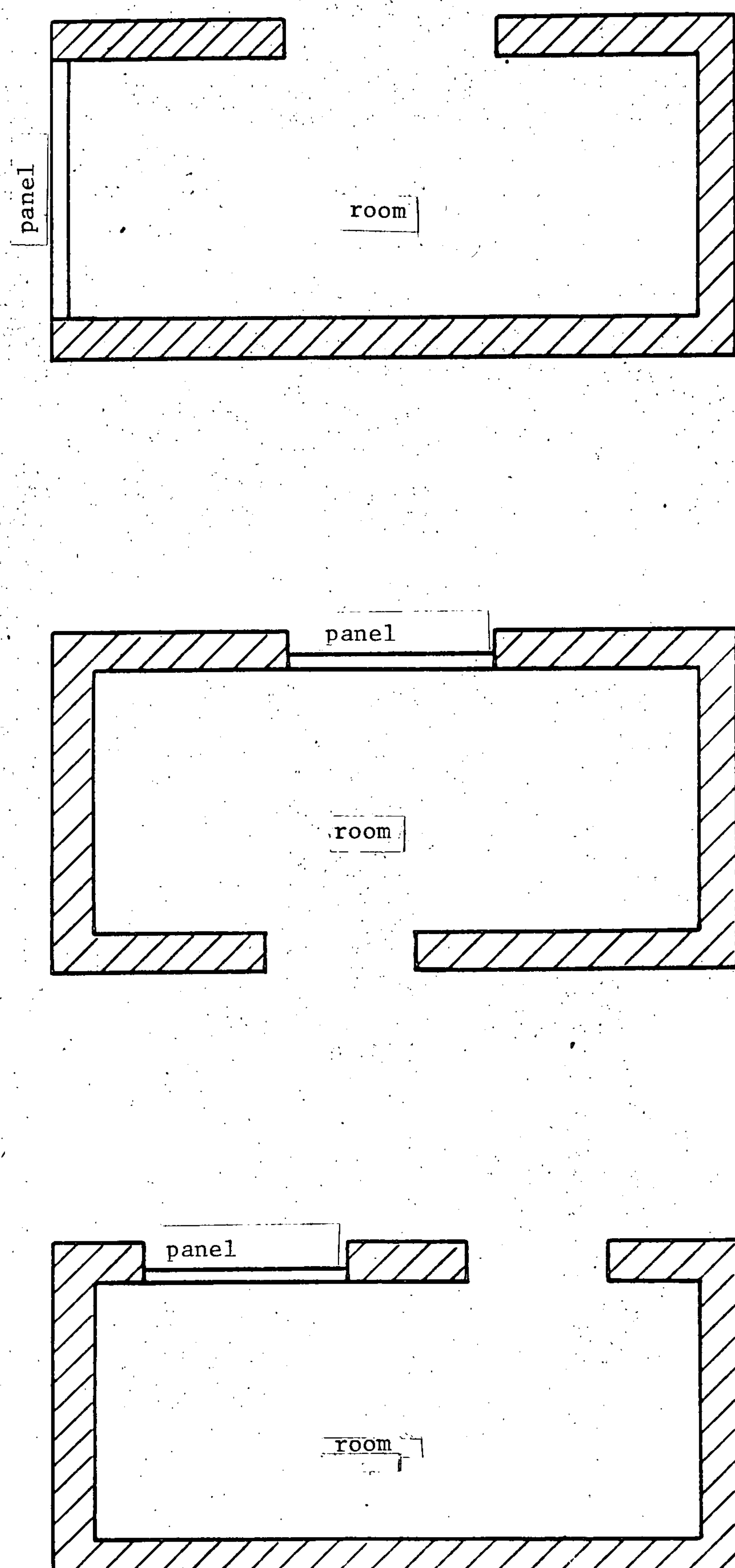

Figure 2.8. Case where the wall is limited by an open room

$$1 - \frac{\omega^2}{\omega_0^2} - \eta \frac{\omega_{11}^2}{\omega_0^2} \frac{\omega^2}{\omega_{11}^2 - \omega^2} = 0$$

If we also take the second mode of the panel ω_{13} into account, the equation is written as:

$$1 - \frac{\omega^2}{\omega_0^2} - \eta \frac{\omega_{11}^2}{\omega_0^2} \left[\frac{\omega^2}{\omega_{11}^2 - \omega^2} + \frac{\omega^2}{9(\omega_{13}^2 - \omega^2)} \right] = 0$$

In order to see the importance of the coupling, we can consider the case $\omega_{11} = \omega_0$, which gives:

- retaining only the first mode of the panel, and by setting $X = \frac{\omega^2}{\omega_0^2}$

$$X^2 - X(2+\eta) + 1 = 0$$

- retaining the two first modes of the panel, and by setting $\lambda = \frac{\omega_{13}^2}{\omega_{11}^2}$ /40

$$-X^3 + X^2\left(2 + \lambda + \frac{10\eta}{2}\right) - X\left(2\lambda + 1 + \frac{\eta}{9} + \eta\lambda\right) + \lambda = 0$$

Figure 2.9 shows these curves for various values λ. They show that they practically all overlap in the vicinity of $X = 1$. The branches of the curves $X < 1$ also all practically overlap.

The transition from λ to ϕ is made using relationship $\lambda = \left(\frac{9+\varphi^2}{1+\varphi^2}\right)^2$. The comparison of Figures 2.7 and 2.10 shows that the coupling becomes greater in the case of the open room considered, as compared with the case where the room is closed. This result was confirmed in [1].

The modes (1.3) are not affected in this way by the coupling for the calculation conditions, except for very large elongations ϕ of the plate (small λ).

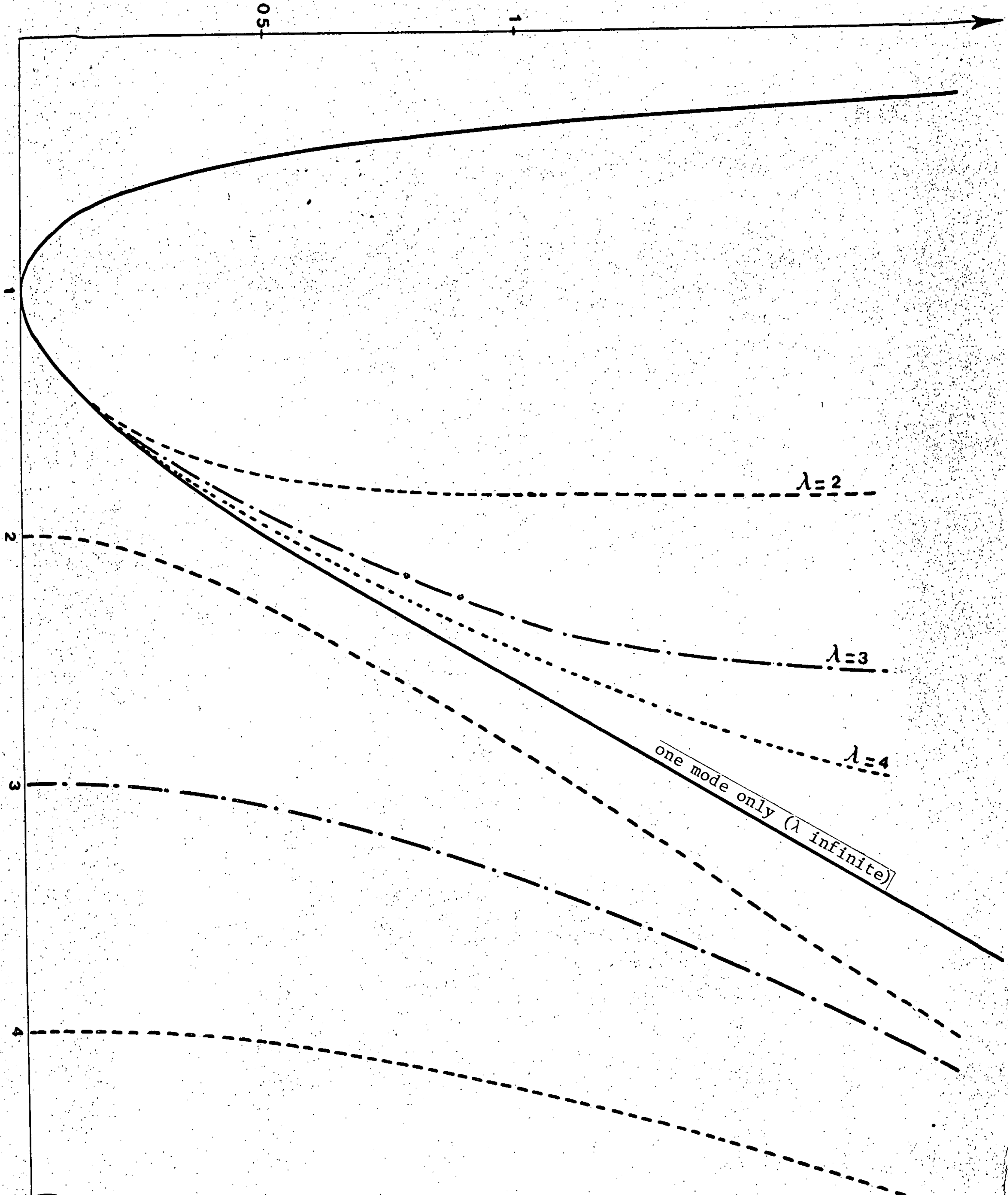

Fig. 2.9. Variation of the eigenpulsations ωr of a simply supported plate as a function of η and λ.

(The room having an opening has a first resonance frequency equal to the first eigenfrequency of the plate)

b) Overpressure of the System Subjected to a Sonic Boom having the Signature $p_e(t)$

Several cases can be considered.

1) The sonic boom impinges only on the opening. This is a case of a sonic boom which penetrates through an open window into a room, one wall of which consists of a light panel, and which is only subjected to the internal pressure in this room. Using Laplace transforms, p_i is given by the following expression, where the damping has not been taken into account:

$$p_i\left[1+\frac{p^2}{\omega_0^2}+\eta\frac{\omega_{11}^2 p^2}{\omega_0^2}\left(\frac{1}{p^2+\omega_{11}^2}+\frac{1}{9(p^2+\omega_{13}^2)}+\dots\right)\right]=p_e$$

In the case mentioned, in general we have: $\omega_0 \leq \omega_{11} \leq \omega_{13}$. Since η is small in general, we can restrict ourselves to considering only the two modes ω_{11} and ω_0.

If we call Ω_0 and Ω_{11} the two first modes of the ensemble "wall-room", we obtain:

$$p_i=\frac{(p^2+\omega_{11}^2)\,\omega_0^2}{(p^2+\Omega_{11}^2)(p^2+\Omega_0^2)}\,p_e$$

or /41

$$p_i=\frac{\omega_0^2}{\Omega_{11}^2-\Omega_0^2}\left[\frac{\Omega_{11}^2-\omega_{11}^2}{p^2+\Omega_{11}^2}+\frac{\omega_{11}^2-\Omega_0^2}{p^2+\Omega_0^2}\right]p_e$$

The oscillations at the frequency Ω_0 are carried out with an amplitude proportional to

$$\frac{\omega_0^2}{\Omega_0^2}\,\frac{\omega_{11}^2-\Omega_0^2}{\Omega_{11}^2-\Omega_0^2}=\frac{\omega_0^2}{\Omega_0^2}\left(1-\frac{\Omega_{11}^2-\omega_{11}^2}{\Omega_{11}^2-\Omega_0^2}\right),$$

and the ones at the frequency ω_0 are carried out at an amplitude proportional to 1. The oscillations at the frequency Ω_{11} are proportional to

$$\frac{\omega_0^2}{\Omega_{11}^2} \quad \frac{\Omega_{11}^2 - \omega_{11}^2}{\Omega_{11}^2 - \Omega_0^2}$$

Since we have $\Omega_{11} > \omega_{11}$, we can assume that the maximum internal overpressure that can be obtained with a sonic boom must diminish with η, if Ω_{11} is sufficiently different from Ω_0. This case $\Omega_{11} \# \Omega_0$ corresponds to a much stronger damping (see § 1.4). The maximum internal overpressure must always diminish with η. (For $\eta \# 0$, this maximum internal overpressure is equal to about twice the overpressure of the crest of the incident sonic boom, according to the first part of this report.)

2) <u>The sonic boom impinges only on the panel</u>

This is the case of a sonic boom which propagates through a closed window into a room having an open door and which communicates with a very large room not subjected to the sonic boom. The equation for p_i is then written as follows, ignoring the damping terms and using the Laplace transforms

$$p_i\left[1 + \frac{p^2}{\omega_0^2} + \frac{\eta\,\omega_{11}^2 p^2}{\omega_0^2}\left(\frac{1}{p^2 + \omega_{11}^2} + \frac{1}{9\,(p^2 + \omega_{13}^2)} + \dots\right)\right]$$
$$= \eta\,\frac{\omega_{11}^2 p^2}{\omega_0^2}\left(\frac{1}{p^2 + \omega_{11}^2} + \frac{1}{9\,(p^2 + \omega_{13}^2)} + \dots\right) p_e$$

for a uniform pressure $p_e(t)$ over the panel.

If we only retain the modes ω_0 and ω_{11}, we obtain:

$$p_i = \frac{\eta\,\omega_{11}^2 p^2}{(p^2 + \Omega_{11}^2)(p^2 + \Omega_0^2)}\, p_e$$

or

$$P_i = \frac{\eta\omega_{11}^2}{\Omega_{11}^2 - \Omega_0^2}\left(\frac{\Omega_{11}^2}{p^2+\Omega_{11}^2} - \frac{\Omega_0^2}{p^2+\Omega_0^2}\right)P_e$$

Depending on the values of Ω_{11}, Ω_0 and the signature interval, we can obtain different combinations of sinusoids, which must agree with the results of [1]. /42

The expression for P_i above shows that the overpressure in the interior will consist of oscillations at the coupling frequency of the resonator and of oscillations of the coupling frequency of the panel. The amplitude of these two types of oscillations will have the same order of magnitude.

In addition, $\eta\frac{\omega_{11}^2}{\Omega_{11}^2 - \Omega_0^2}$ can become very large, if Ω_{11} is not too far separated from Ω_0 (which assumes that η is not too large). Under these conditions, a considerable internal overpressure p_i can be expected, because the overpressure radiated by the window is essentially in resonance with the Helholtz resonator.

A more complete study of the internal overpressure should be carried out, taking the dampings into account.

3) <u>The sonic boom impinges on the panel and the opening</u>

This is the case of a sonic boom which penetrates into a closed room through an open window and a closed window. These two openings are sufficiently far away from each other so that there will be no interaction.

The solution for P_i is obtained by adding the two previous solutions:

$$\frac{P_i}{P_e} = \frac{1}{\Omega_{11}^2 - \Omega_0^2}\left[\frac{\omega_0^2(\Omega_{11}^2 - \omega_{11}^2) + \eta\omega_{11}^2\Omega_{11}^2}{p^2+\Omega_{11}^2} + \frac{\omega_0^2(\omega_{11}^2 - \Omega_0^2) - \eta\omega_{11}^2\Omega_0^2}{p^2+\Omega_0^2}\right]$$

If the coupling η is large enough, and if the two modes Ω_0 and Ω_{11} are highly excited, the addition of these two pressures obtained in the two first cases will result in an amplification of the internal overpressure which is larger than is obtained without coupling. Craggs [1] theoretically obtained an amplification of approximately 2.7 (without damping) in a particular case which was not selected in advance.

c) Response of the Panel /43

We will only retain the pulsations ω_{11} and ω_0, for the same case as for the calculation of the overpressure.

1) The sonic boom impinges only on the opening

From § 1.4, we find:

$$\begin{cases} (p^2+\omega_{11}^2)\varphi_{11} = \dfrac{16}{\pi^2 \rho_p h} P_i \\ P_i = -\dfrac{4ab\rho c^2}{V\pi^2 \omega_0^2} \dfrac{p^2}{1+\dfrac{p^2}{\omega_0^2}} \varphi_{11} + \dfrac{\omega_0^2 P_e}{p^2+\omega_0^2} \end{cases}$$

which results in:

$$\varphi_{11} = \frac{16\,\omega_0^2}{\pi^2 \rho_p h} \frac{P_e}{(p^2+\Omega_{11}^2)(p^2+\Omega_0^2)}$$

If there were no coupling, the internal pressure would be given by

$$P_{oi} = \frac{\omega_0^2 P_e}{p^2+\omega_0^2}$$

and the displacement would be:

$$\varphi_{11} = \frac{16}{\pi^2 \rho_p h} \frac{P_{oi}}{(p^2+\omega_{11}^2)} = \frac{16\,\omega_0^2}{\pi^2 \rho_p h} \frac{P_e}{(p^2+\omega_{11}^2)(p^2+\omega_0^2)}$$

Everything behaves as though the panel were responding to an internal pressure equal to $\frac{\omega_0^2}{\Omega_0^2} \frac{\Omega_0^2 P_e}{p^2+\Omega_0^2}$, the maximum of which is greater than the value of P_i calculated without coupling.

It is difficult to predict a priori if the coupling increases or does not increase the response of the panel. In any case, with a very loose coupling, we can easily predict a dynamic amplification factor in this case equal to five times the dynamic amplification factor which would be obtained considering the panel subjected to a sonic boom (the dynamic amplification factor of thepanel subjected to a damped sinusoid is two to three, and the maximum internal overpressure equals one to two times the overpressure of the crest of the sonic boom).

2) <u>The sonic boom impinges only on the panel</u> /44

According to § 1.4 the equations are written as:

$$(p^2+\omega_{11}^2)\varphi_{11} = \frac{16}{\pi^2 \rho_p h}(P_i - P_e)$$

$$P_i = -\frac{4ab\rho c^2}{\pi^2 V \omega_0^2} \frac{p^2}{1+\frac{p^2}{\omega_0^2}} \varphi_{11}$$

from which:

$$\varphi_{11} = -\frac{16}{\pi^2 \rho_p h} \frac{(p^2+\omega_0^2)\quad P_e}{(p^2+\Omega_0^2)(p^2+\Omega_{11}^2)}$$

The coupling is important for a light panel, which must have a low frequency, and we can assume that $\Omega_{11} \leq \Omega_0$, which leads to the first case of b), by inverting the roles of Ω_{11} and Ω_0.

$$\varphi_{11} = -\frac{16}{\pi^2 \rho_p h (\Omega_0^2 - \Omega_{11}^2)} \left[\frac{\Omega_0^2 - \omega_0^2}{p^2 + \Omega_0^2} + \frac{\omega_0^2 - \Omega_{11}^2}{p^2 + \Omega_{11}^2} \right] P_e$$

In the same way, the displacement of the panel consists of oscillations at the frequencies $\frac{\Omega_0}{2\pi}$ and $\frac{\Omega_{11}}{2\pi}$, (the oscillations at the frequency $\frac{\Omega_0}{2\pi}$ vanish for a negligible amount of coupling).

The oscillations at the frequency $\frac{\Omega_{11}}{2\pi}$ must oppose the oscillations at the frequency $\frac{\Omega_0}{2\pi}$ in general, and the displacements obtained must be smaller than when there is coupling.

3) <u>The sonic boom impinges on the opening and the panel</u>

The displacement is obtained by adding the two preceding displacements:

$$\varphi_{11} = -\frac{16}{\pi^2 \rho_p h (\Omega_{11}^2 - \Omega_0^2)} \left[\frac{\Omega_{11}^2}{p^2 + \Omega_{11}^2} - \frac{\Omega_0^2}{p^2 + \Omega_0^2} \right] P_e$$

The panel (assumed to have the eigenpulsation Ω_{11}), would have a response equal to the following, if it were not subjected to the sonic boom:

$$\varphi_{110} = -\frac{16\ P_e}{\pi^2 \rho_p h (p^2 + \Omega_{11}^2)}$$

The difference $Q_{11} - Q_{110}$ equals:

$$\varphi_{11} - \varphi_{110} = \frac{16\,\Omega_0^2\, P_e}{\pi^2 \rho_p h\,(p^2+\Omega_{11}^2)(p^2+\Omega_0^2)}$$

This expression shows that the displacement due to the internal pressure is increased. This displacement can equal five times the static displacement which would result from a uniform pressure in the room, equal to the maximum value of p_e.

The displacement of the panel must therefore become quite large and considerably exceed the displacement which is predicted by considering the hypothesis of a plate subjected to a ballistic detonation.

This amplification will be attenuated as the coupling increases. The results obtained are not contrary to those obtained by Craggs [1].

2 .4 CONCLUSIONS

a) Results

The classical theory of vibrations of a homogeneous plate has been reviewed. It makes it possible to predict the maximum responses of a plate subjected to a sonic boom for the simple cases, using the curves of the dynamic amplification factor.

This theory led to a simplified approach to the influence of rooms adjacent to the wall, taking into account the existing couplings. Using the hypotheses made, it seemed that a ballistic detonation penetrating into a room through an opening (window or open door) and through a light panel can bring about large overpressures and displacements. A large bay containing glass panels (showcase) installed in an opening will result in an internal overpressure the maximum amplitude of which can be equal to 2 to 3 times the overpressure of the incident crests. This effect is even greater than in the case of a partition. This maximum amplitude will be only slightly damped, which will create a psychological stress. On the other hand, the displacements of this bay to the outside can considerably exceed the ones which are predicted if we consider the bay only subjected to the sonic boom (more than 5 times using the hypotheses made). /46

b) Criticism

1) When the displacements of a panel become large (larger than the thickness), it is necessary to take the membrane stresses into account in the vibration equation of the plate. These stresses can considerably reduce the displacements (see [6]).

The study of the vibration of glass plates in the vicinity of the rupture point cannot be made using the adopted hypotheses.

2) Numerous panels currently in use in construction are not homogeneous. The fact that several materials are used can lead to additional stresses.

3) The aging of the buildings produced additional settling and additional stresses, which are translated into internal prestresses.

These should be taken into account in a study of the vibrations and stresses in a plate.

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Translated for National Aeronautics and Space Administration under contract No. NASw 2035, by SCITRAN, P. O. Box 5456, Santa Barbara, California, 93108